Aberration Theory

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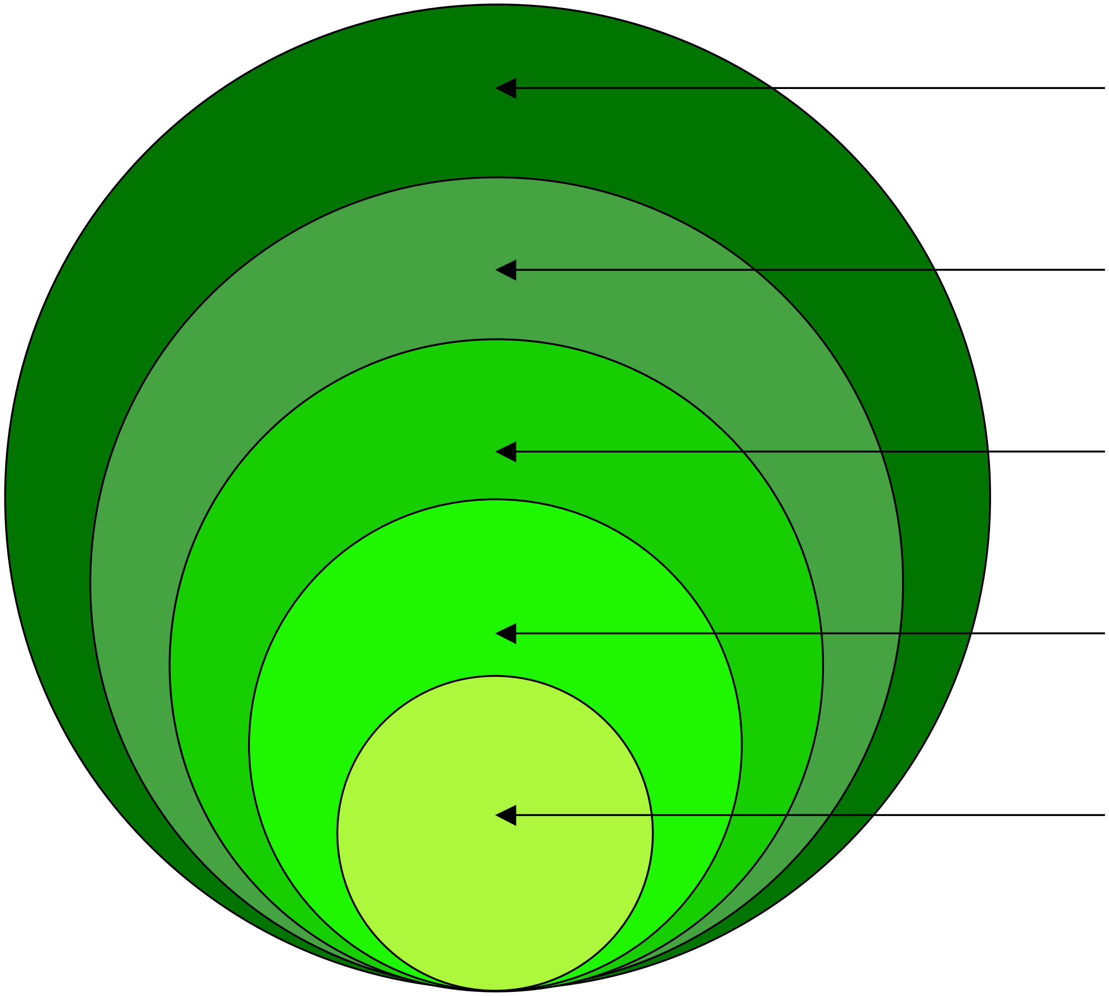
Optics



Quantum Optics

Coherent Optics

Diffractive Optics
(Fourier Optics)

Geometrical Optics
(Aberration theory)

Paraxial Optics
(First Order Optics)
(Gaussian Optics)

Outline

- ***What Is Wavefront?***
  - **Huygens's principle, Snell's law, Fermat's principle**
  - **Paraxial (first order) approximation**

- ***What Kind Of Wavefront Aberrations Are There?***
  - **Monochromatic aberration (Seidel and wave aberrations)**
  - **Chromatic aberration (Longitudinal, Transverse)**

- ***Why Are These Aberrations Important?***
  - **Relationship between aberrations and image quality**
  - **(Pupil function, PSF, MTF, Image convolution…)**

- ***How Can We Measure These Aberrations Of The Eye?***
  - **Different types of wavefront sensors**

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What Is Wavefront?

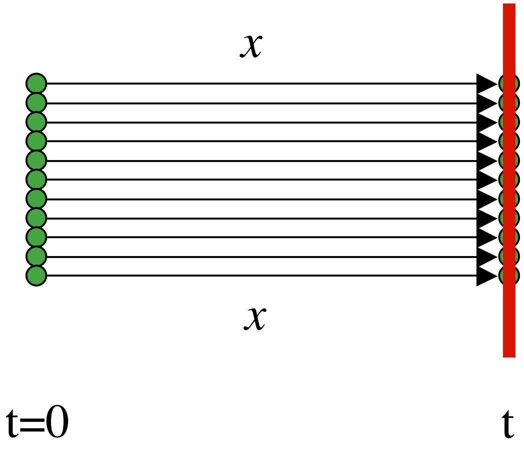
Huygens’s principle
Snell’s law
Fermat’s principle
Paraxial (first order) approximation

Wavefront vs Ray

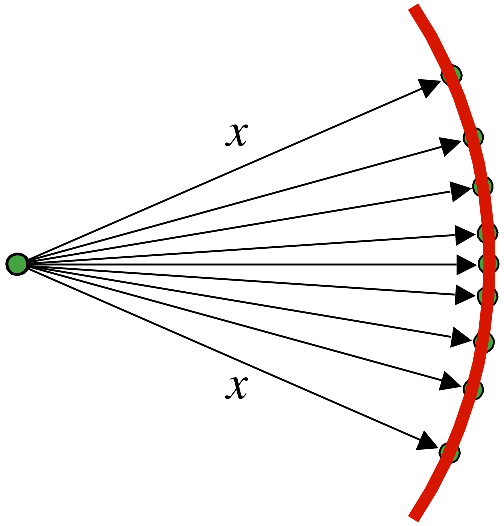
"A wavefront is a surface over which an optical disturbance has a constant phase."

Harmonic wave function $$\psi(x,t) = A\sin(\underbrace{kx-\omega t}_{\text{Phase}})$$

Plane wavefront

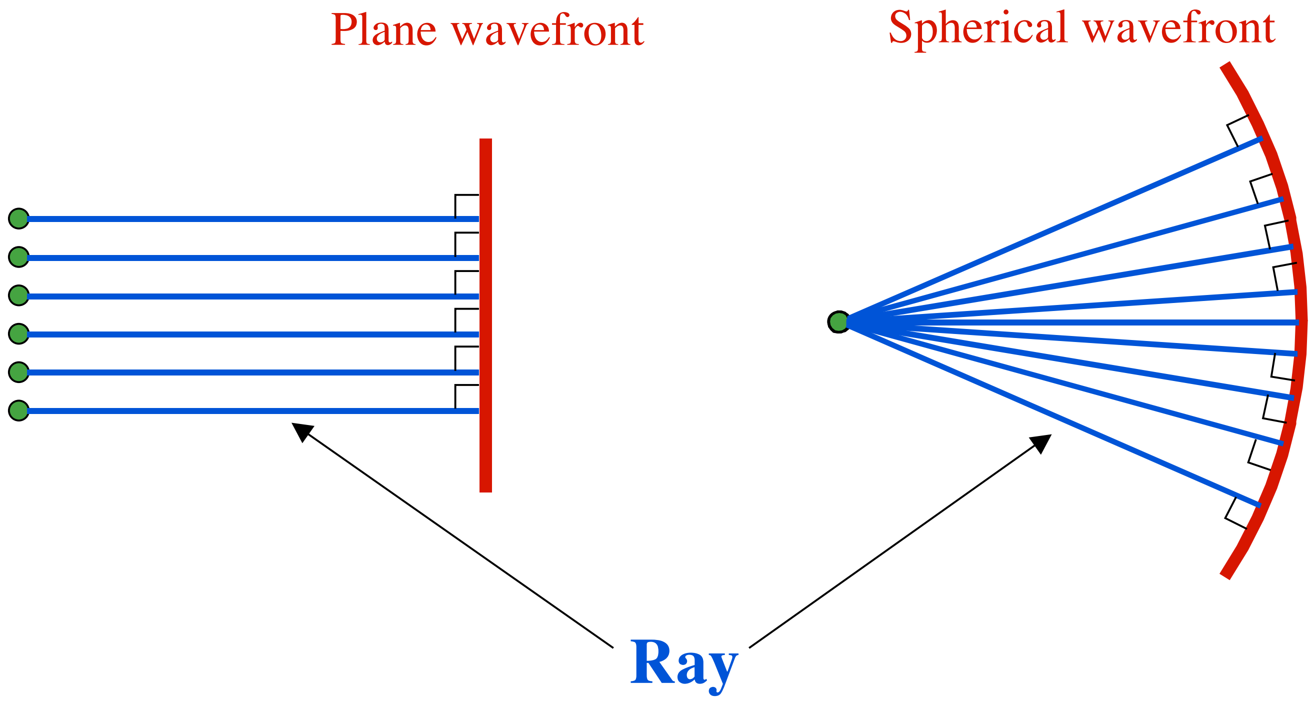


Spherical wavefront



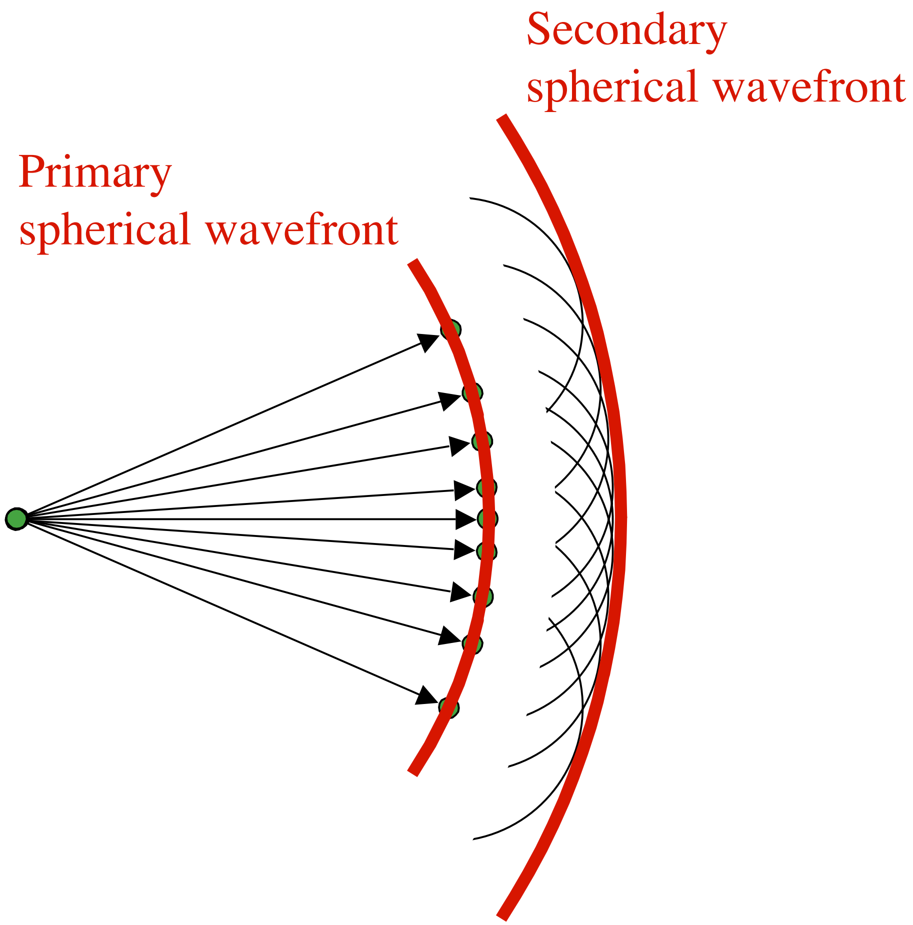
Wavefront vs Ray

"Rays are lines normal to the wavefronts at every point of intersection."

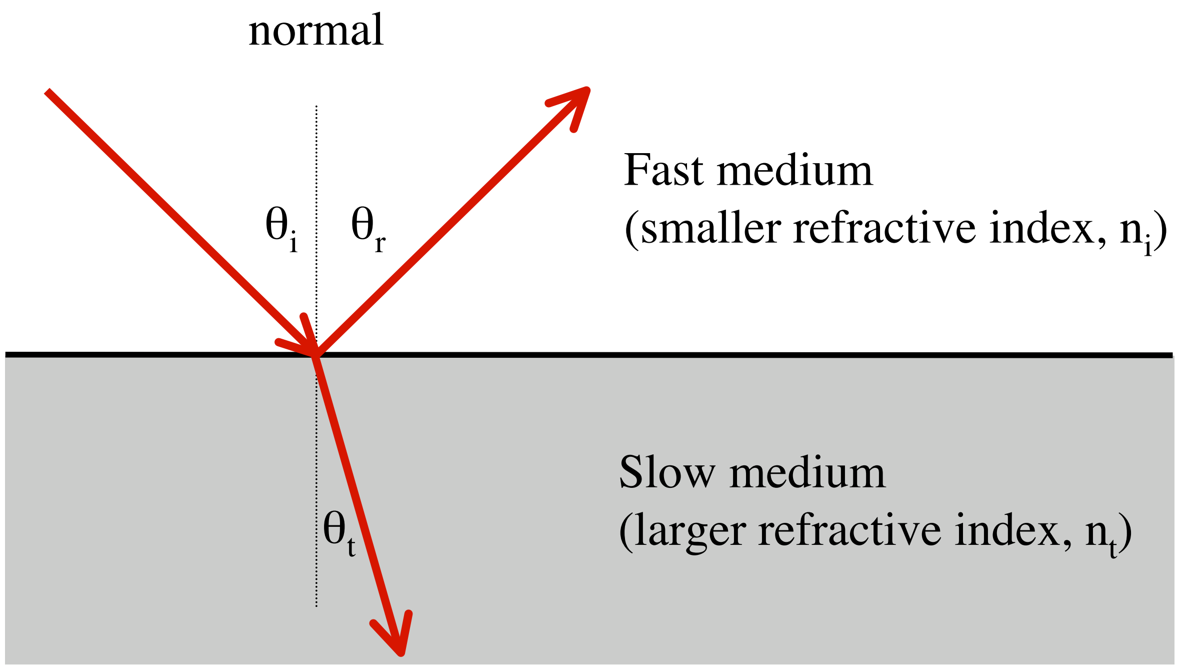


Huygens's Principle

"Every point on a primary wavefront serves as the source of spherical secondary wavelets, such that the primary wavefront at some later time is the envelope of these wavelets."



Snell’s law



Fast medium
(smaller refractive index, n_i)

Slow medium
(larger refractive index, n_t)

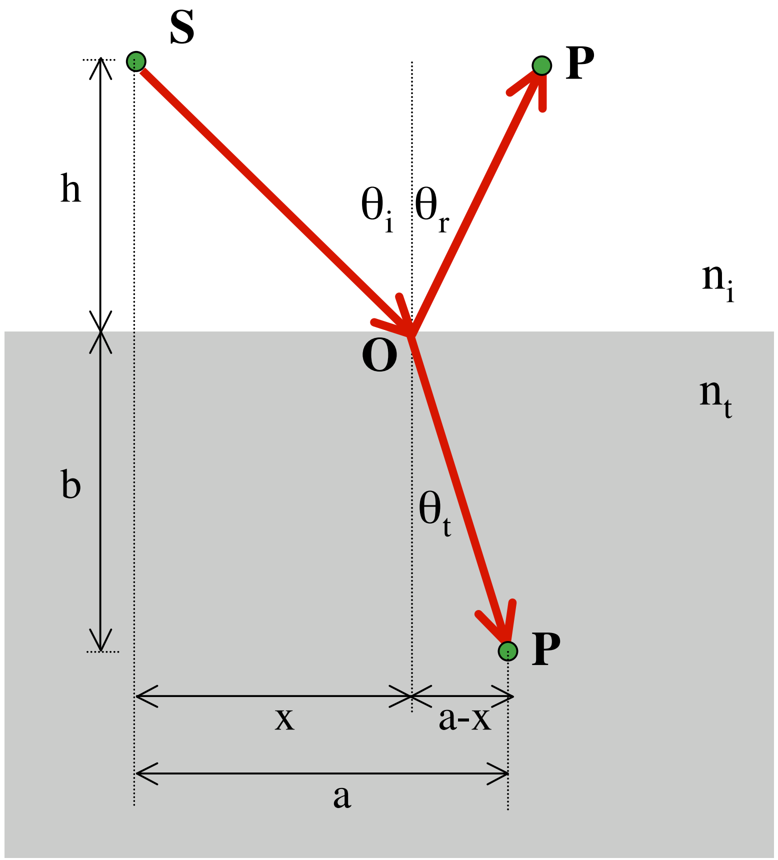
$$\frac{n_i}{n_t} = \frac{\sin(\theta_t)}{\sin(\theta_i)}$$

Reflection: $\theta_i = \theta_r$

Refraction: $\theta_i > \theta_t$ when $n_i < n_t$

Fermat’s Principle

“The path actually taken by light in going from some point S to a point P is the shortest optical path length (OPL).”



$$OPL = n_i \cdot \overline{SO} + n_t \cdot \overline{OP}$$

$$= n_i \cdot \sqrt{h^2 + x^2} + n_t \cdot \sqrt{b^2 + (a-x)^2}$$

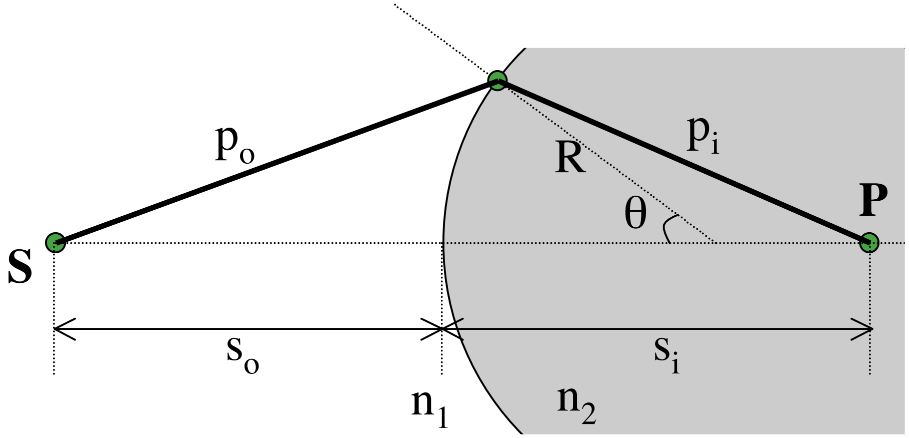
$$\frac{\mathrm{d}OPL}{\mathrm{d}x} = 0 \text{ to minimize } OPL$$

$$n_i \cdot \frac{x}{\sqrt{h^2 + x^2}} + n_t \cdot \frac{-(a-x)}{\sqrt{b^2 + (a-x)^2}} = 0$$

$$\Downarrow$$

$$\frac{n_i}{n_t} = \frac{\sin(\theta_t)}{\sin(\theta_i)}$$

Paraxial Optics (First order optics)



$$\frac{n_1 R(s_o + R)\sin\theta}{p_o} = \frac{n_2 R(s_i - R)\sin\theta}{p_i}$$

Approximation

$$\sin\theta \approx \theta$$

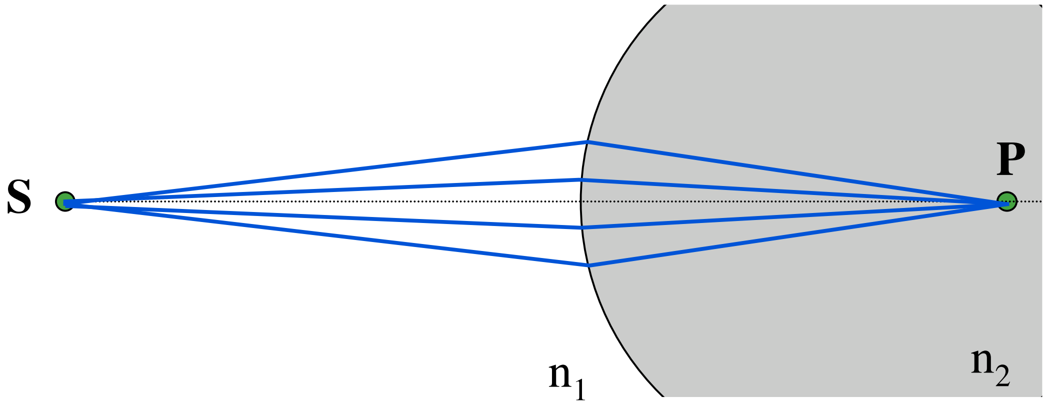
$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

Lens maker's formula

Paraxial Optics (First order optics)

“The emerging wavefront segment corresponding to these paraxial rays is essentially spherical and will form a ”perfect” image at its center P.”



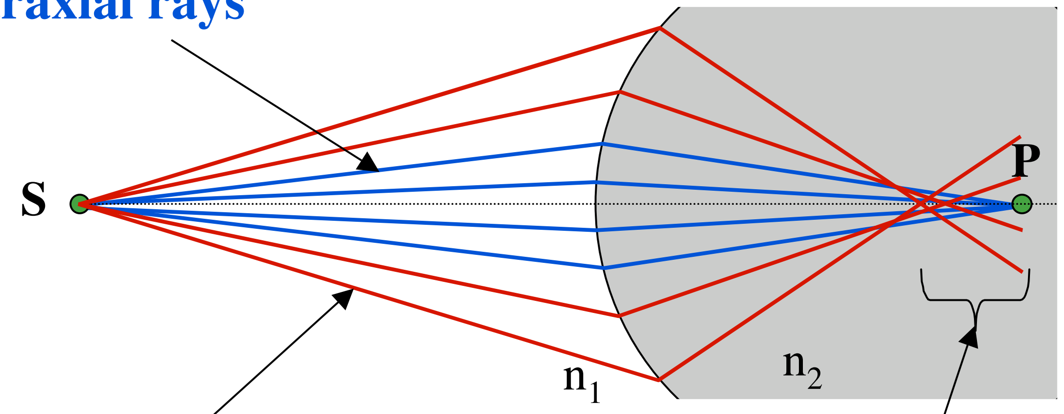
Third Order Optics

“The paraxial approximation, sin_ ≈ _, is somewhat unsatisfactory if rays from the periphery of a lens are considered.”

$$\sin\theta = \theta \boxed{- \frac{\theta^3}{3!}}$$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} + \boxed{h^2\left[\frac{n_1}{2s_o}\left(\frac{1}{s_o} + \frac{1}{R}\right)^2 + \frac{n_2}{2s_i}\left(\frac{1}{R} - \frac{1}{s_i}\right)^2\right]}$$

Paraxial rays



Perpheral rays

Aberrations

What Kinds Of Wavefront Aberrations Are There?

Monochromatic aberration
(Seidel and wave aberrations)

Chromatic aberration
(Longitudinal, Transverse)

Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration

- Coma

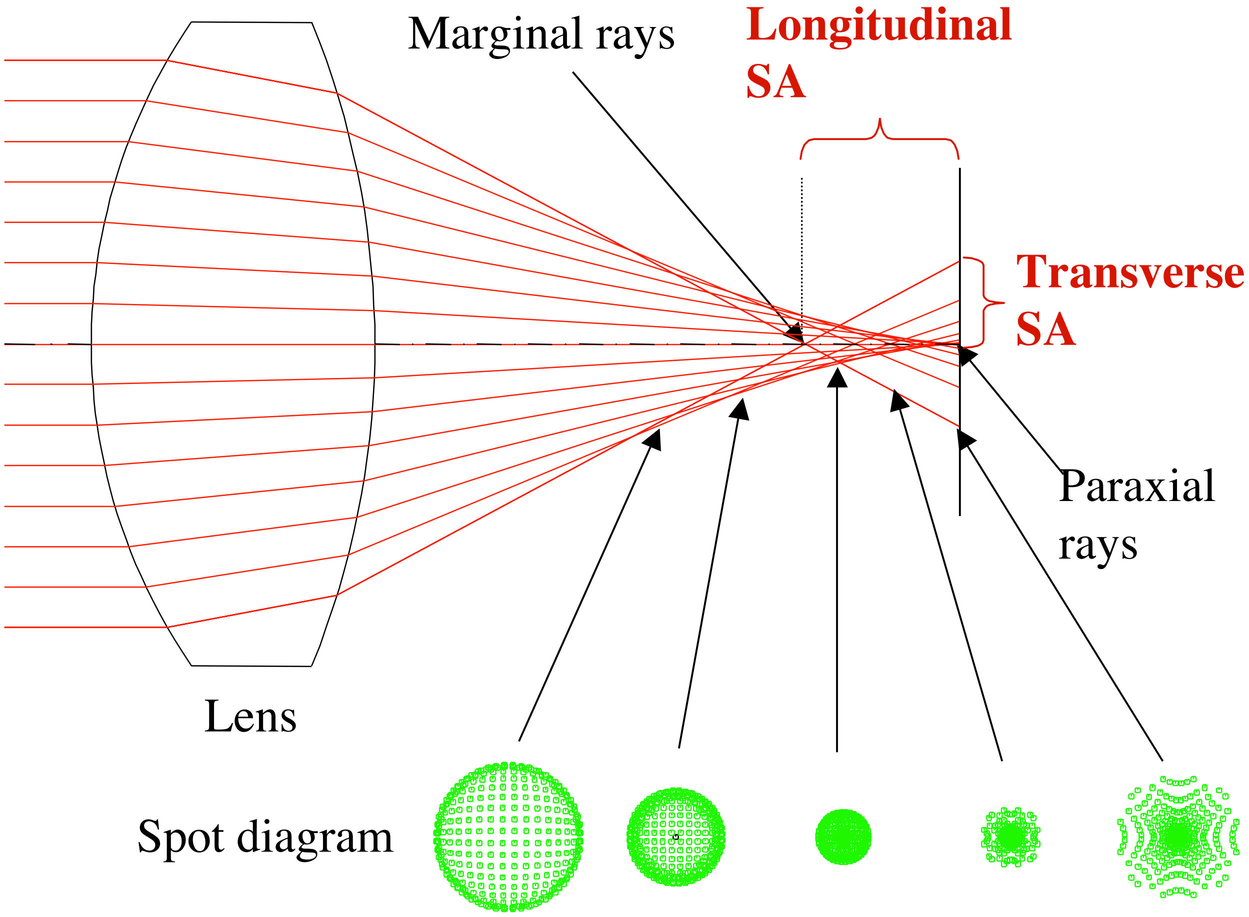
- Astigmatism

- Field Curvature

- Distortion

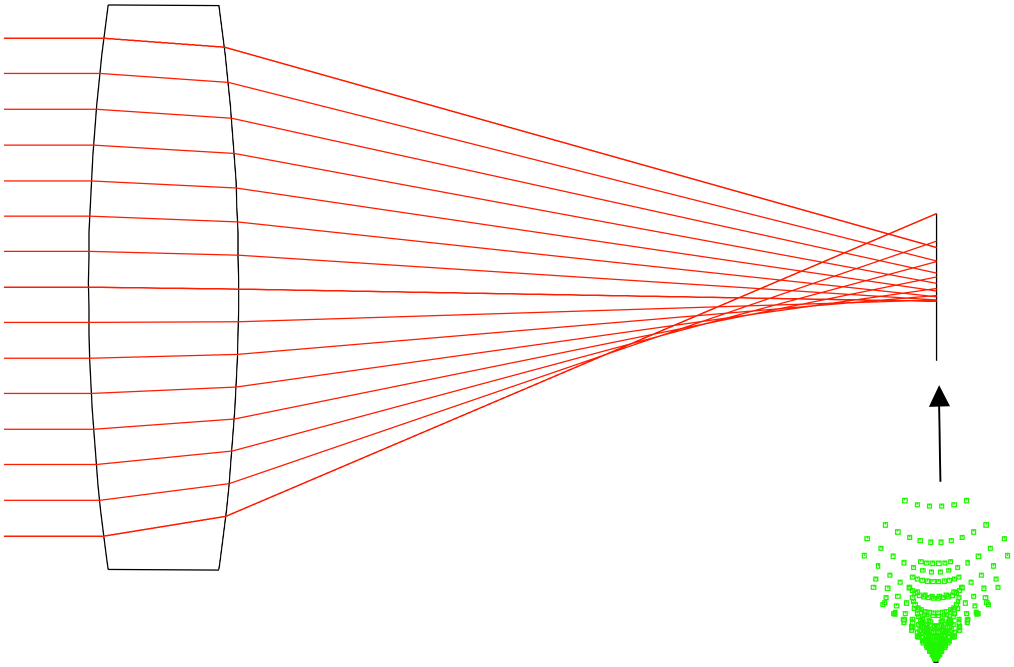
Monochromatic aberrations (Seidel aberrations)

- **Spherical Aberration**
- Coma
- Astigmatism
- Field Curvature
- Distortion



Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration
- **Coma**
- Astigmatism
- Field Curvature
- Distortion



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- **Astigmatism**
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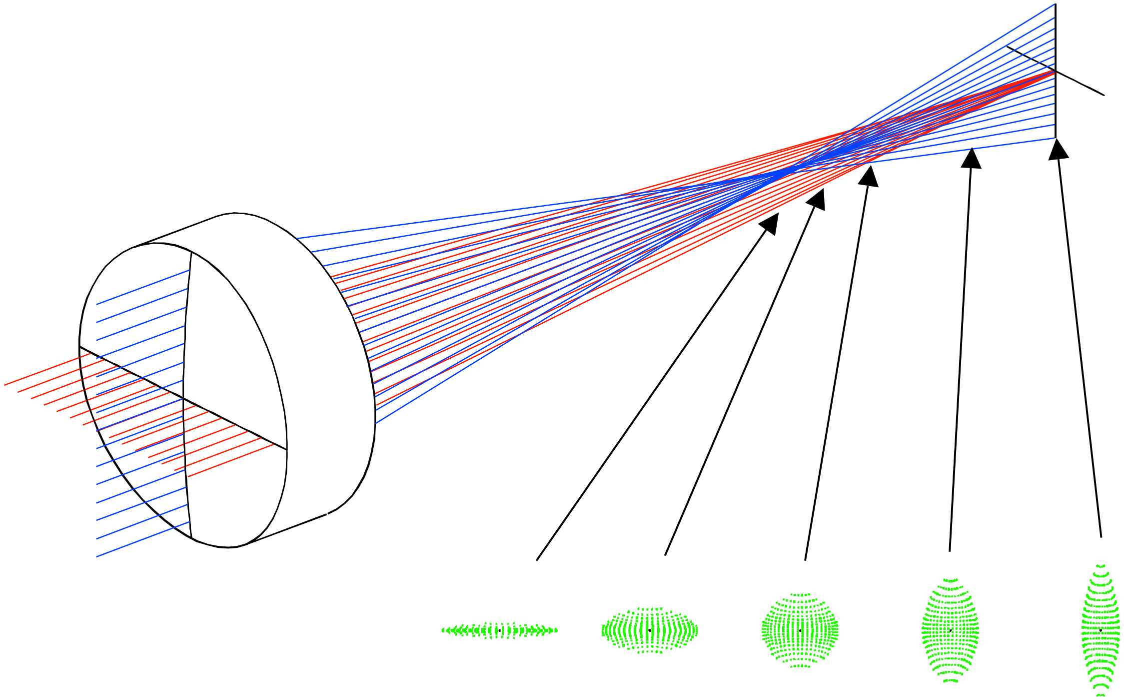
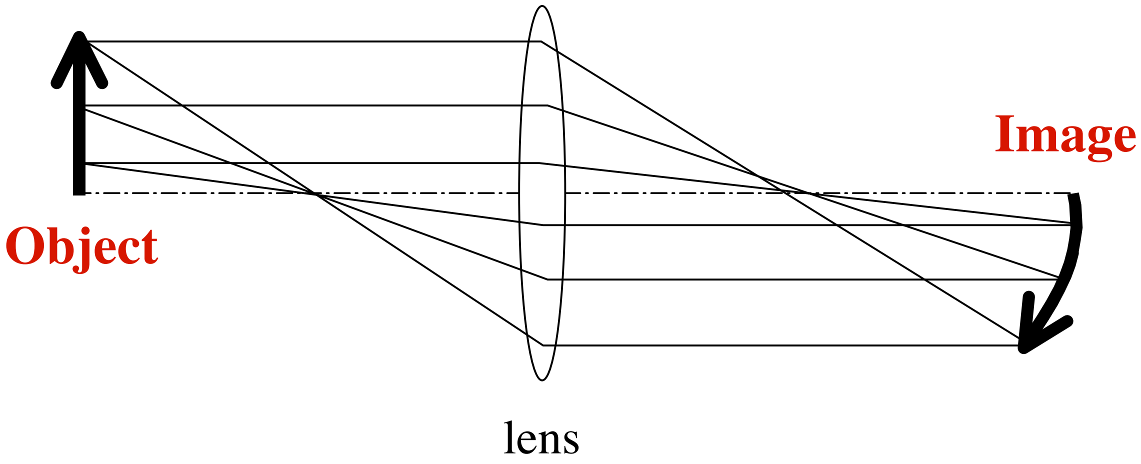
- Distortion



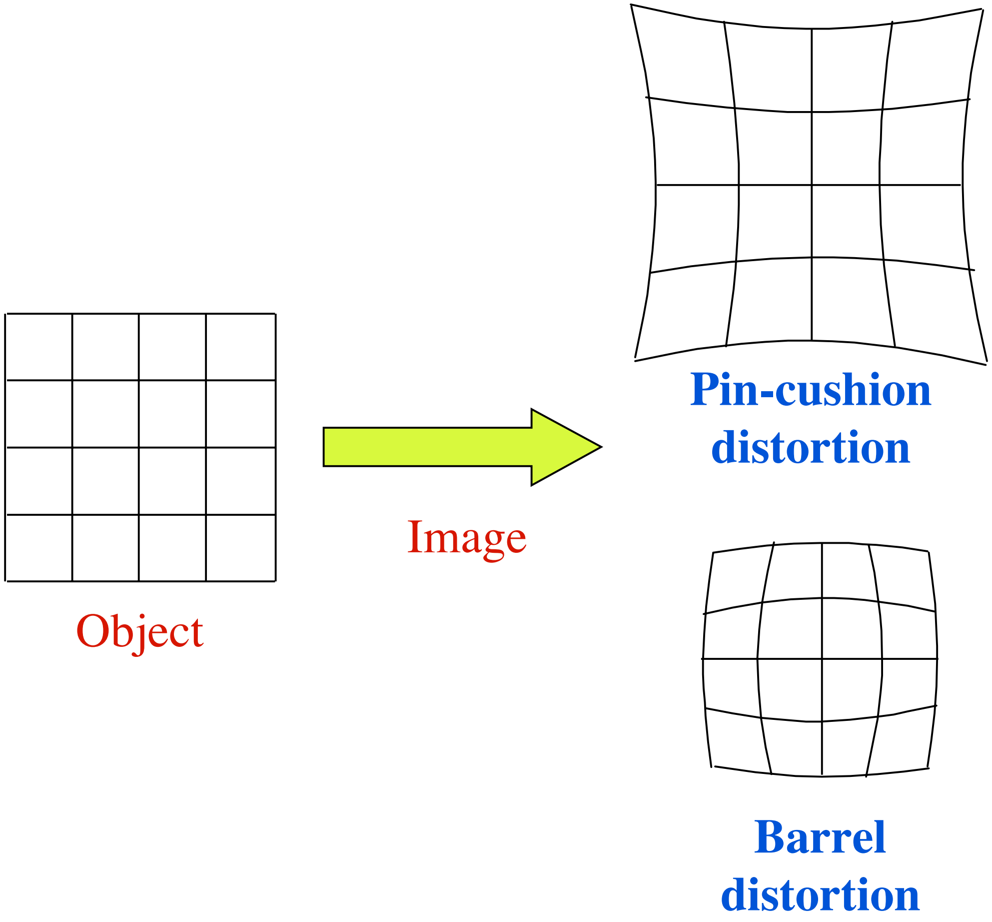
Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration
- Coma
- Astigmatism
- **Field Curvature**
- Distortion



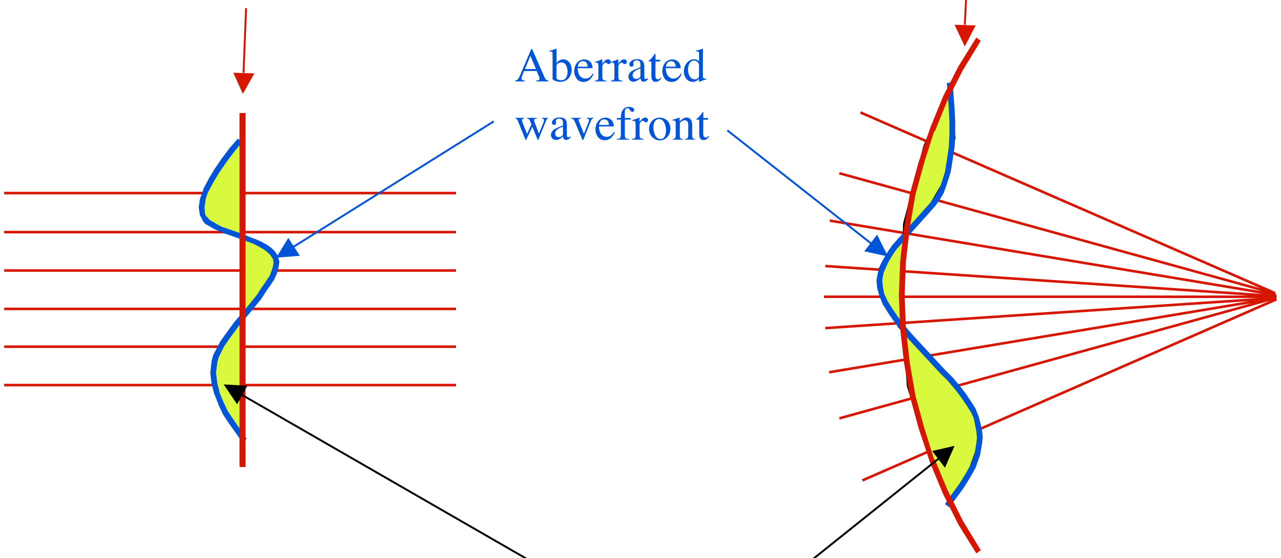
Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration
- Coma
- Astigmatism
- Field Curvature
- **Distortion**



Monochromatic aberrations (wave aberrations)

“The optical deviations of the wavefront from a reference plane or spherical wavefront.”



Reference
plane wavefront

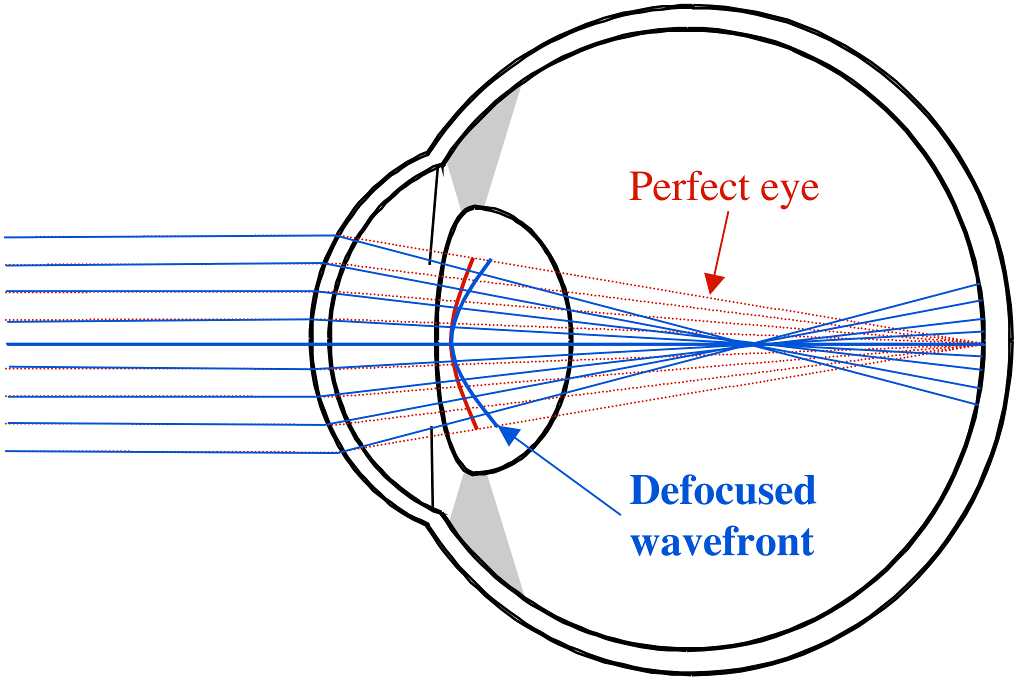
Reference
spherical wavefront

Aberrated
wavefront

Wave aberration

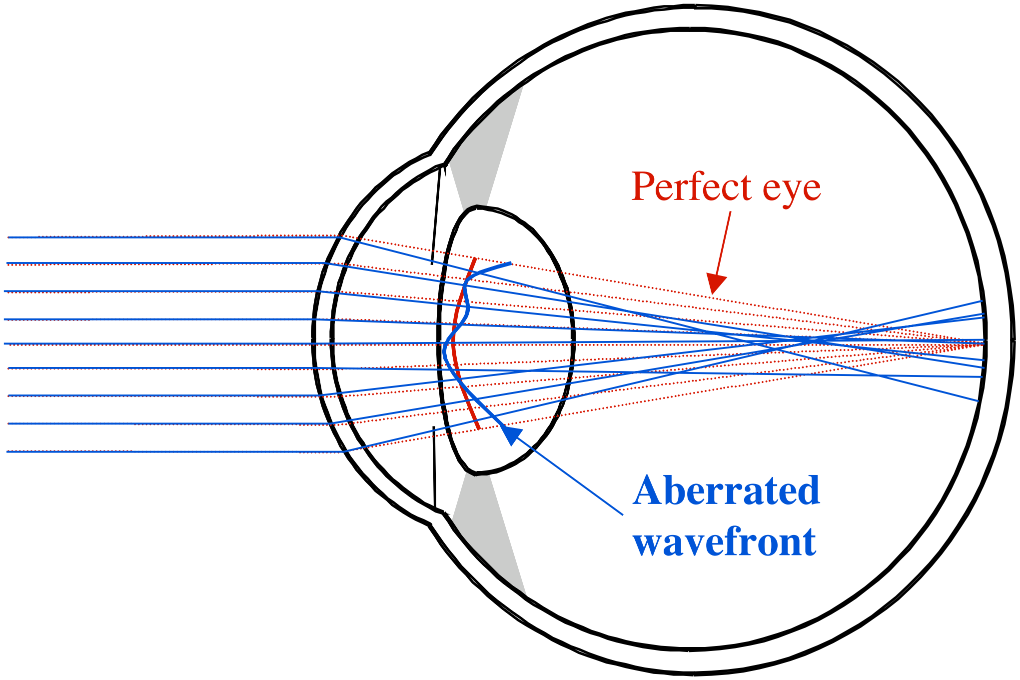
Wave aberrations (defocus)

Myopic (near sighted) eye

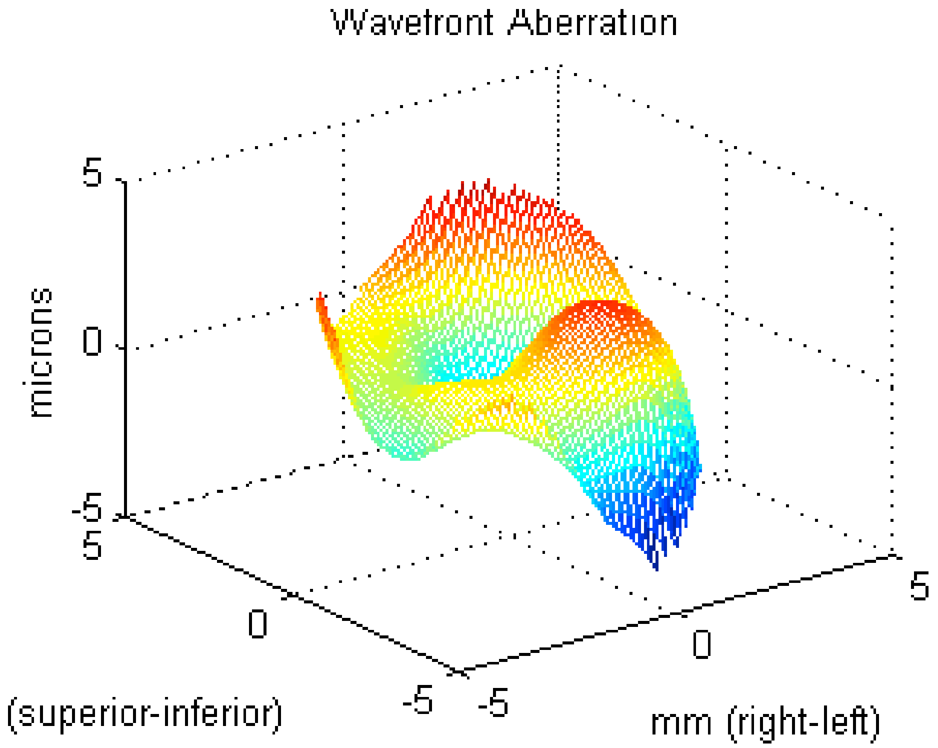


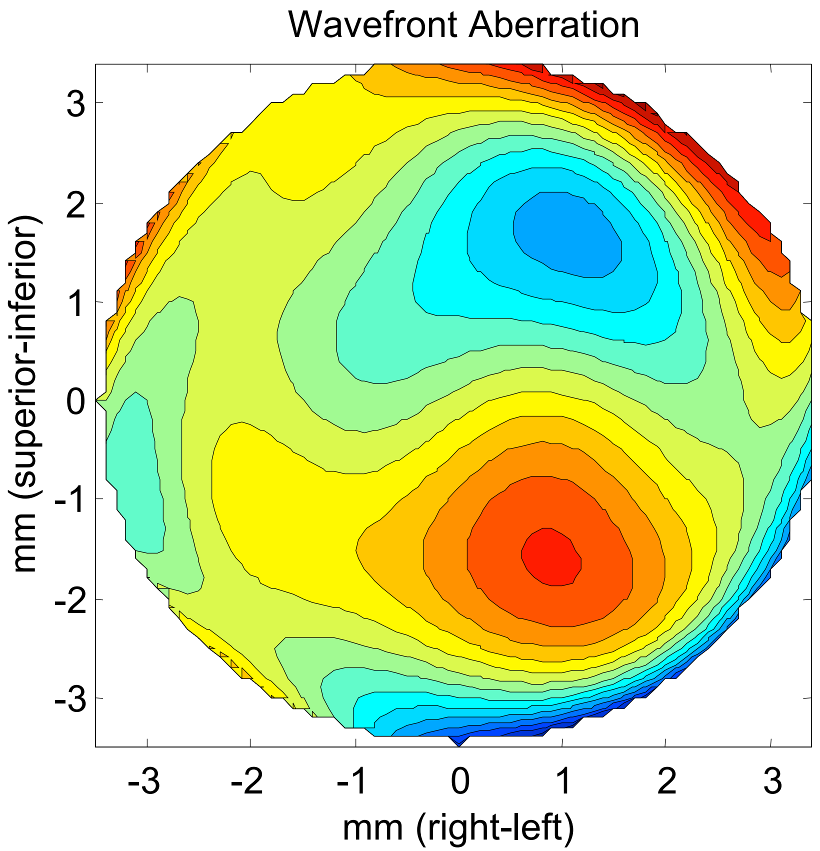
Wave aberrations (higher order)

Eye with higher order aberrations



Wave Aberration of a Surface





Mathematical description of the aberration.

Zernike circle polynomials

$$W(\_,\theta) = \sum C_n^m \, Z_n^m(\_,\theta)$$

Wavefront
aberration

Zernike
coefficient

Zernike polynomials
(wavefront mode)

m: angular frequency

astigmatism

$$Z_2^2(\_,\theta) = \_^2 \cos 2\theta$$

n: radial order

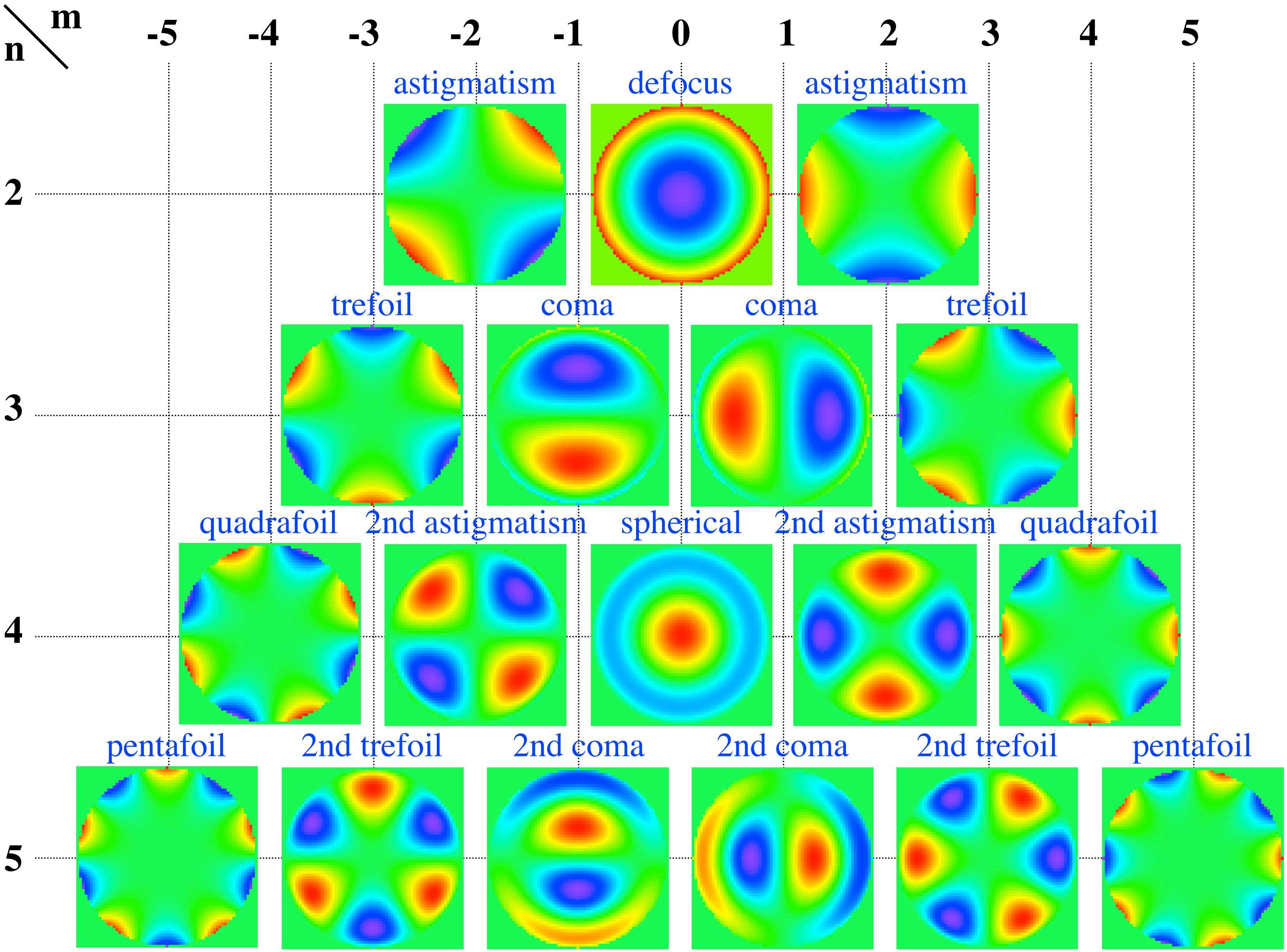
Zernike polynomials

n = order	m = frequency	$Z_n^m(\rho,\theta)$
0	0	1
1	-1	$2\ \rho \sin\theta$
1	1	$2\ \rho \cos\theta$
2	-2	$\sqrt{6}\,\rho^2 \sin 2\theta$
2	0	$\sqrt{3}\,(2\rho^2-1)$
2	2	$\sqrt{6}\,\rho^2 \cos 2\theta$
3	-3	$\sqrt{8}\,\rho^3 \sin 3\theta$
3	-1	$\sqrt{8}\,(3\rho^3-2\rho) \sin\theta$
3	1	$\sqrt{8}\,(3\rho^3-2\rho) \cos\theta$
3	3	$\sqrt{8}\,\rho^3 \cos 3\theta$
4	-4	$\sqrt{10}\,\rho^4 \sin 4\theta$
4	-2	$\sqrt{10}\,(4\rho^4-3\rho^2) \sin 2\theta$
4	0	$\sqrt{5}\,(6\rho^4-6\rho^2+1)$
4	2	$\sqrt{10}\,(4\rho^4-3\rho^2) \cos 2\theta$
4	4	$\sqrt{10}\,\rho^4 \cos 4\theta$
5	-5	$\sqrt{12}\,\rho^5 \sin 5\theta$
5	-3	$\sqrt{12}\,(5\rho^5-4\rho^3) \sin 3\theta$
5	-1	$\sqrt{12}\,(10\rho^5-12\rho^3+3\rho) \sin\theta$
5	1	$\sqrt{12}\,(10\rho^5-12\rho^3+3\rho) \cos\theta$
5	3	$\sqrt{12}\,(5\rho^5-4\rho^3) \cos 3\theta$

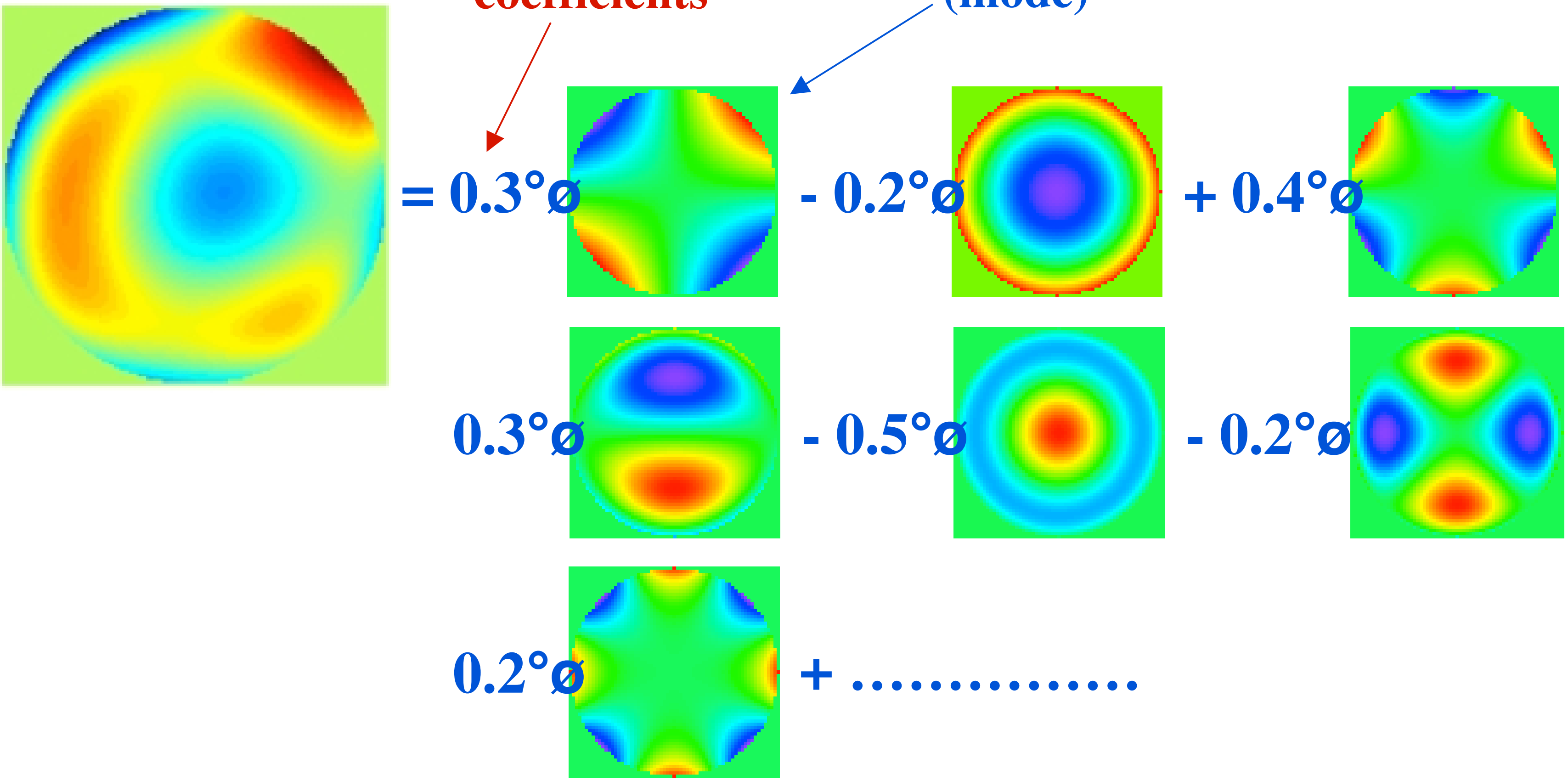
Second order aberrations

Higher order aberrations

Wavefront mode for each Zernike polynomial



Wavefront aberration and Zernike coefficients



Wave aberration

Zernike coefficients

Zernike polynomial (mode)

= 0.3°ø - 0.2°ø + 0.4°ø +

0.3°ø - 0.5°ø - 0.2°ø +

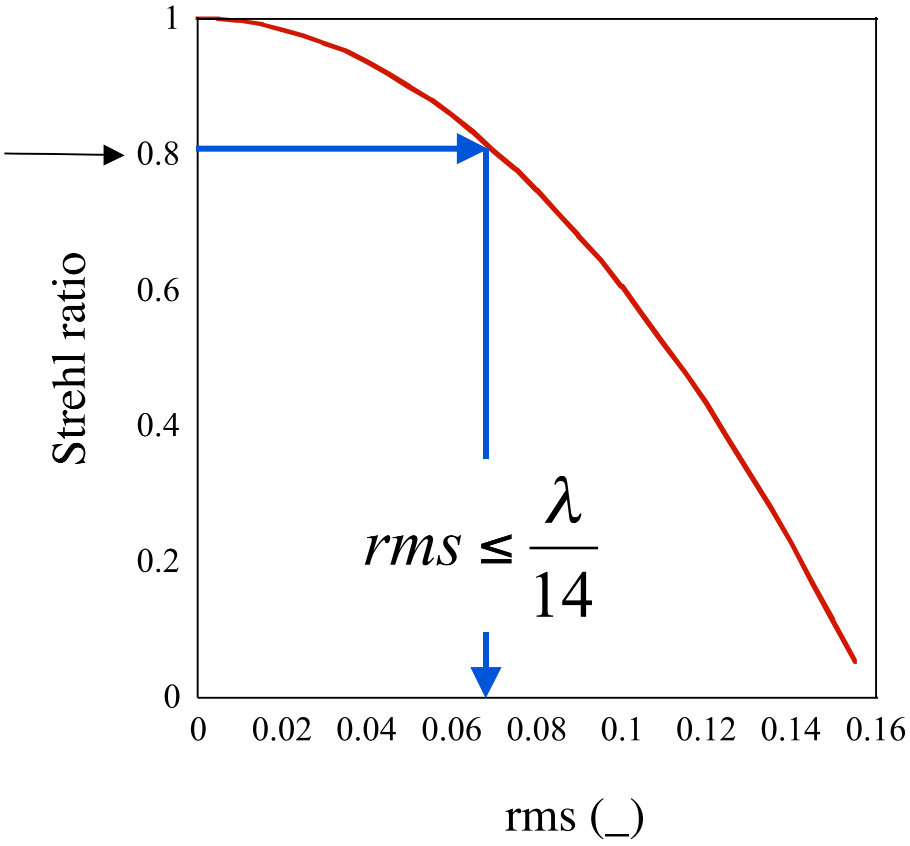
0.2°ø + ……………

Wave aberration ⇄ ***Zernike coefficients***

Wavefront rms error

$$rms = \sqrt{\sum \left(C_n^m\right)^2} \quad \Longrightarrow \quad Strehl \approx 1 - \left(\frac{2\pi}{\lambda}\right)^2 rms^2$$

when *rms* is small.



Diffraction-limited
(Strehl ≥ 0.8)

$$rms \le \frac{\lambda}{14}$$

Rayleigh's _/4 rule

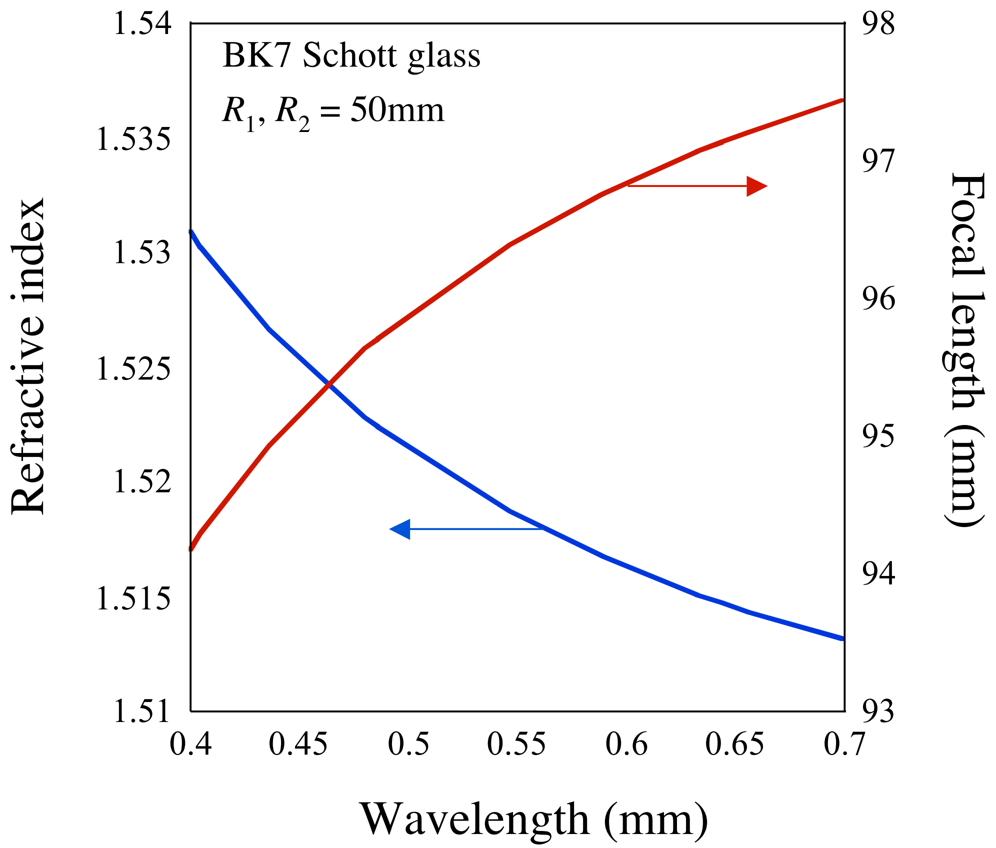
$$W_{p-v} \le \frac{\lambda}{4}$$

Chromatic Aberration

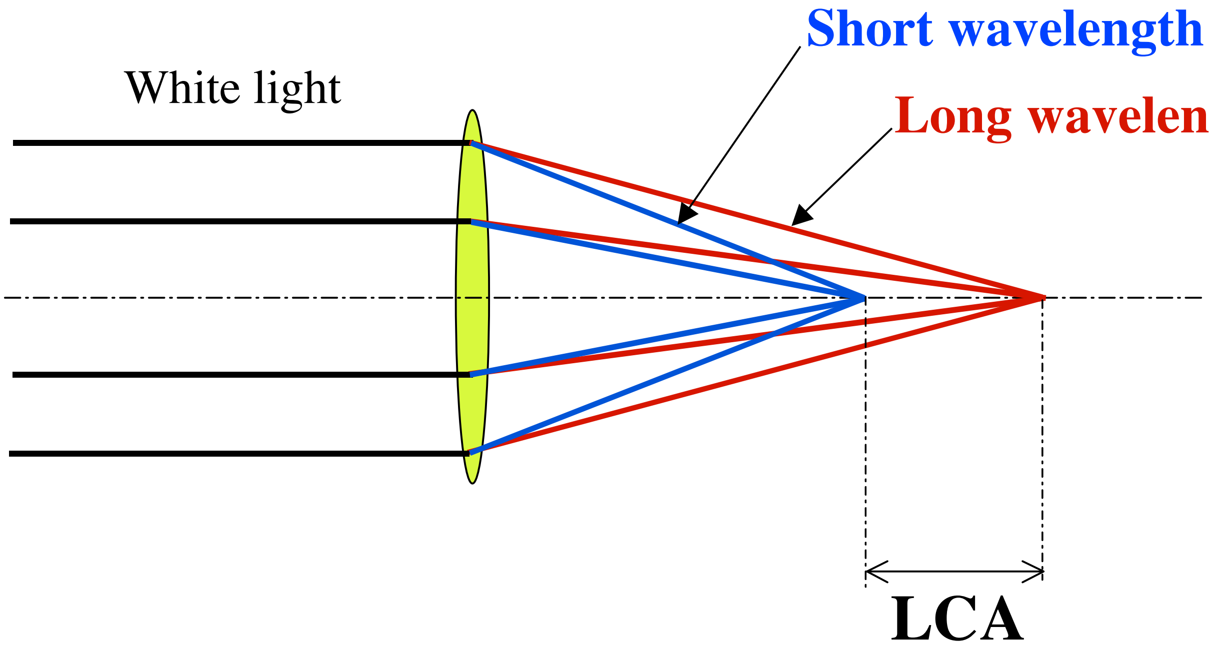
Lensmaker's formula:

$$\frac{1}{f} = (n-1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

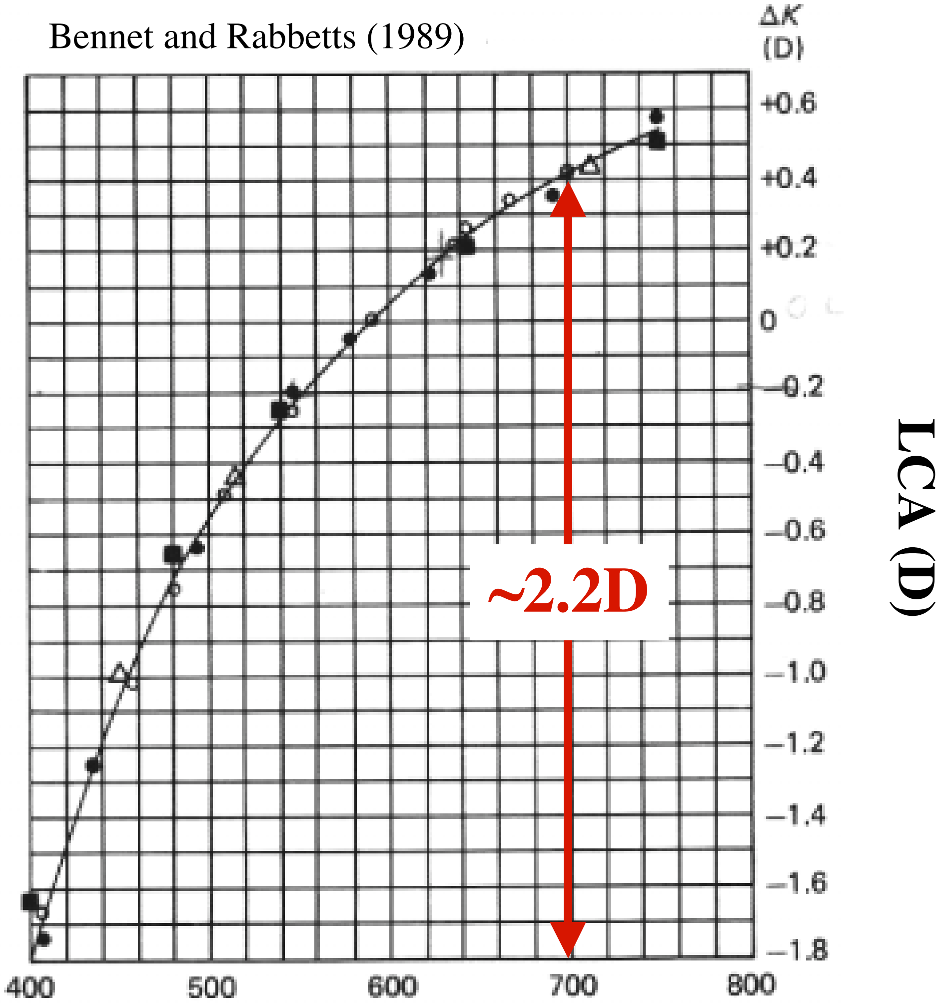
$n = n(\_) \rightarrow f = f\,(\_)$ **for polychromatic light**



Longitudinal (axial) chromatic aberration (LCA)



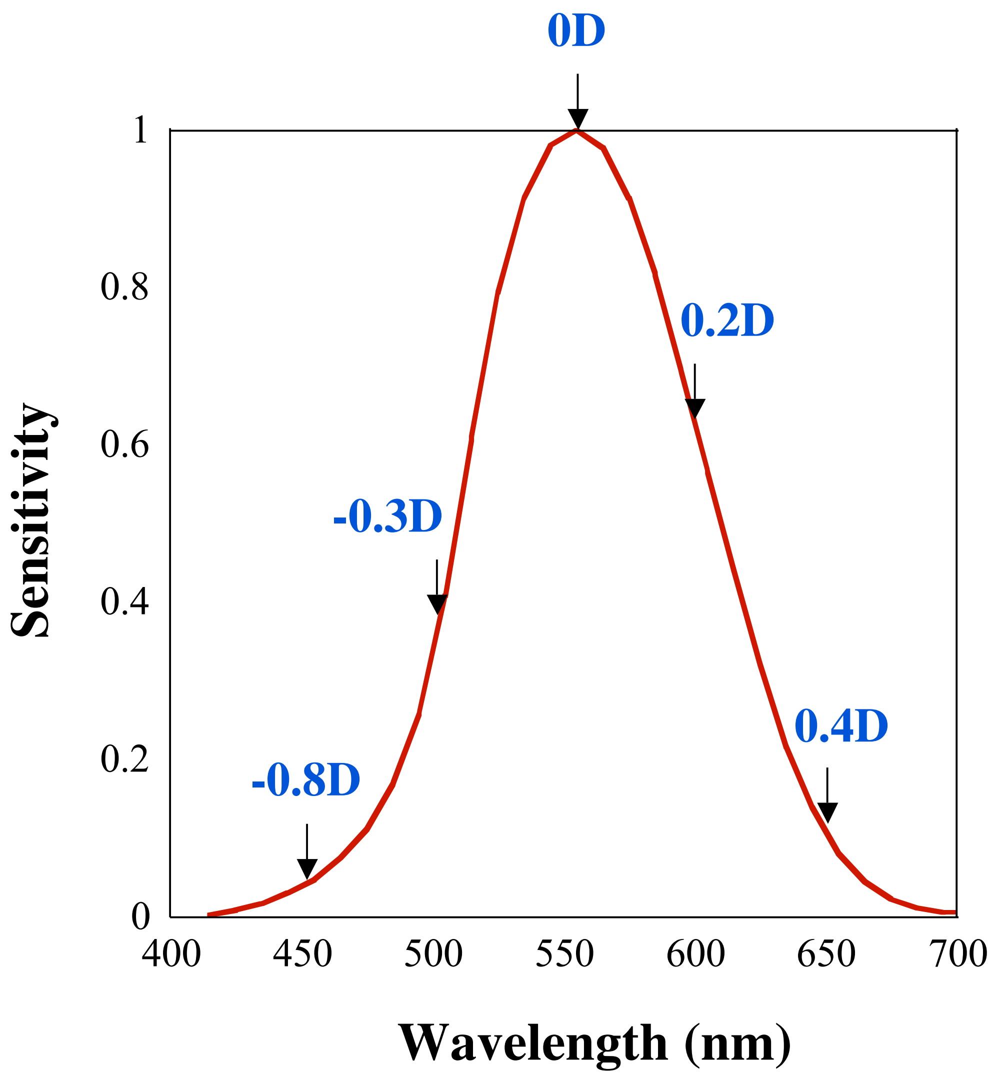
LCA of the human eye



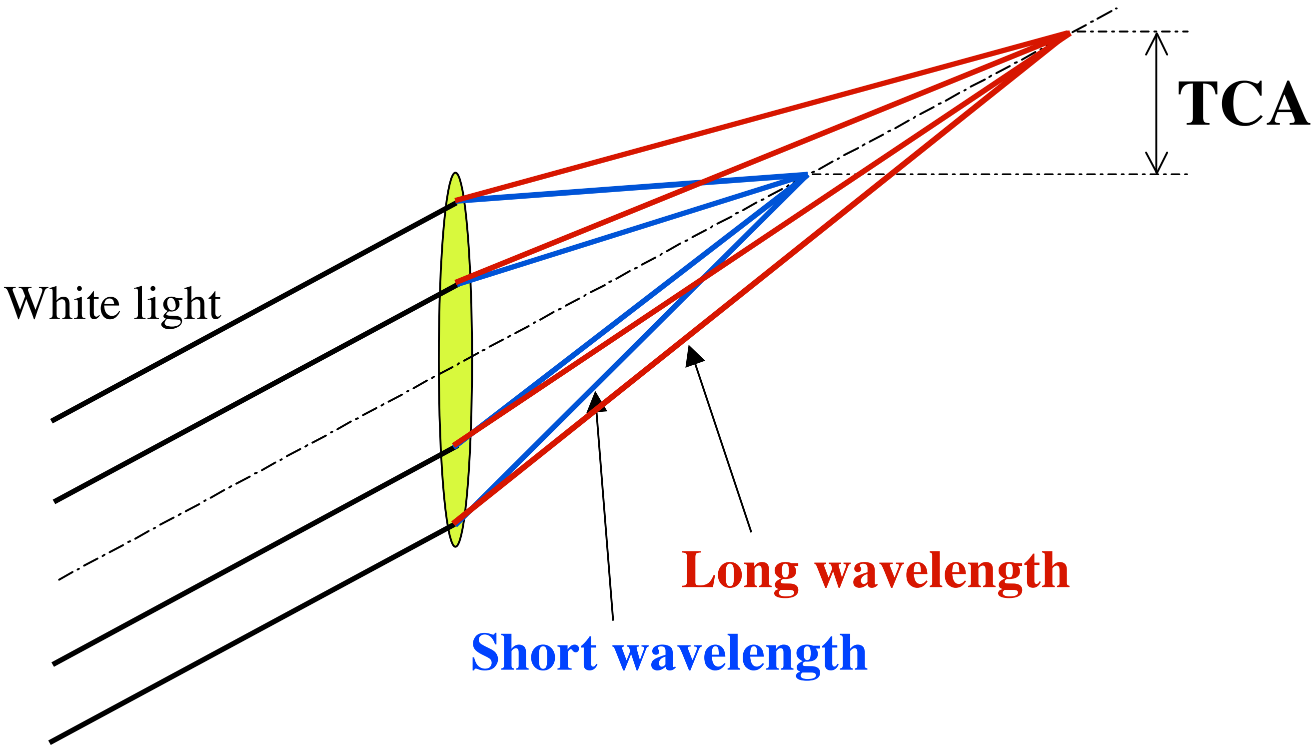
Wavelength (nm)

Spectral sensitivity of the human eye

Optical effect of eye's LCA on image quality ≈ 0.2D defocus for monochromatic light



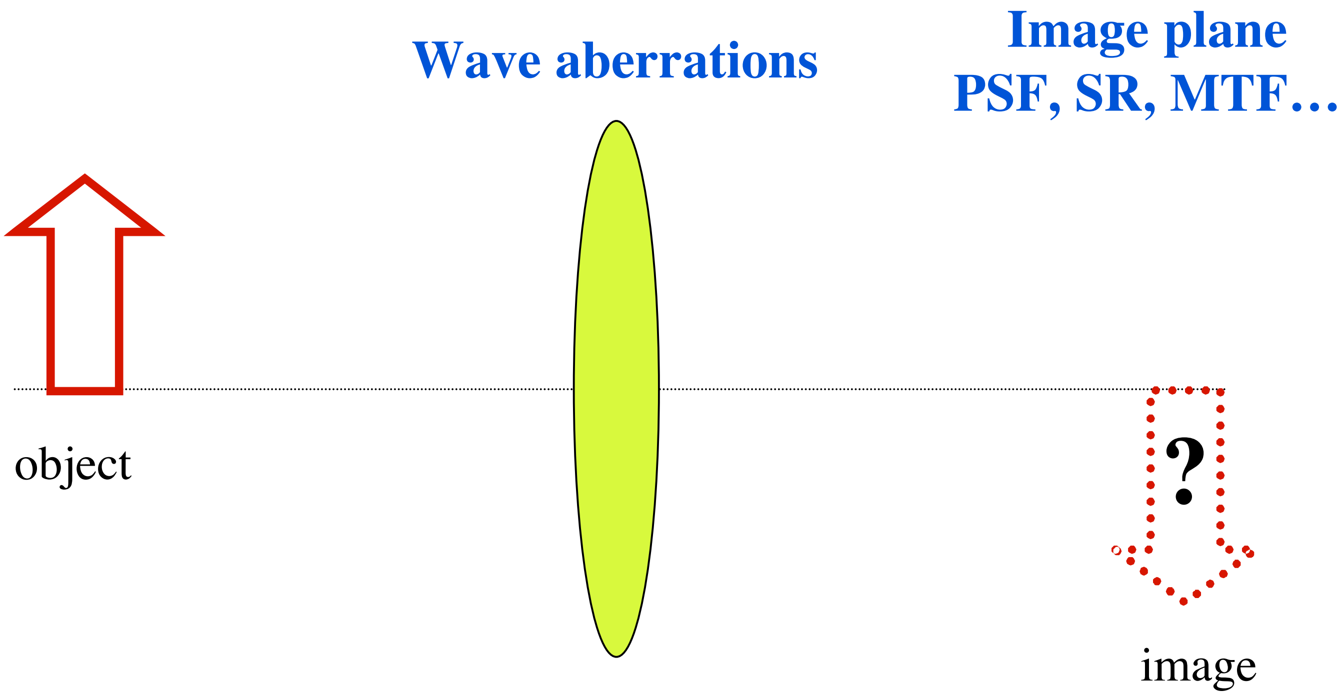
Transverse (lateral) chromatic aberration (TCA)



Why Are These Aberrations Important?

Relationship between aberrations and image quality
(Pupil function, Rms, PSF, SR, OTF,
MTF, PTF, Image convolution…)

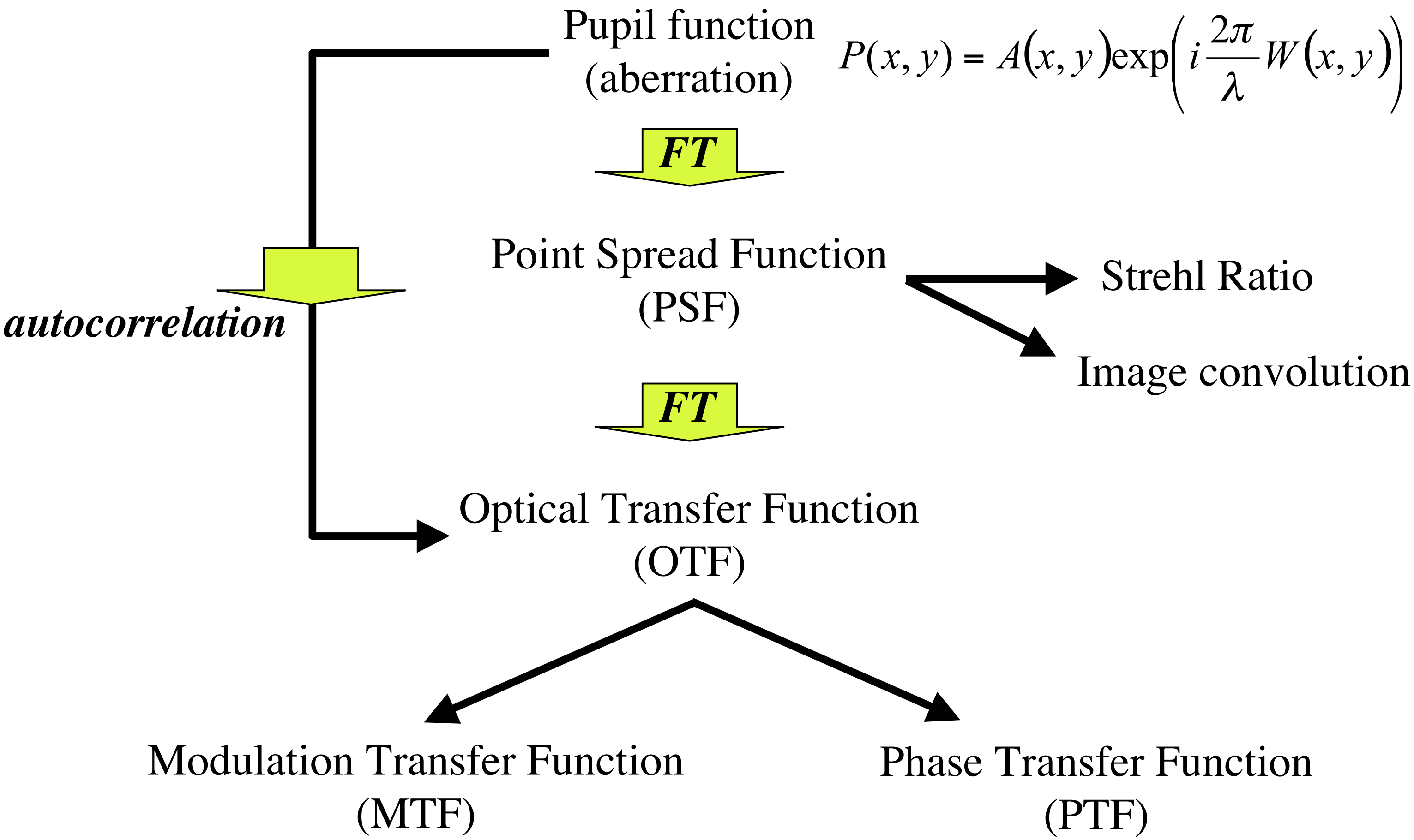
Image quality



How well can an optical system form image?

We can improve image quality by correcting the aberrations.

Aberration vs Image quality



Pupil function
(aberration)

$$P(x,y) = A(x,y)\exp\left(i\frac{2\pi}{\lambda}W(x,y)\right)$$

FT

autocorrelation

Point Spread Function
(PSF)

Strehl Ratio

Image convolution

FT

Optical Transfer Function
(OTF)

Modulation Transfer Function
(MTF)

Phase Transfer Function
(PTF)

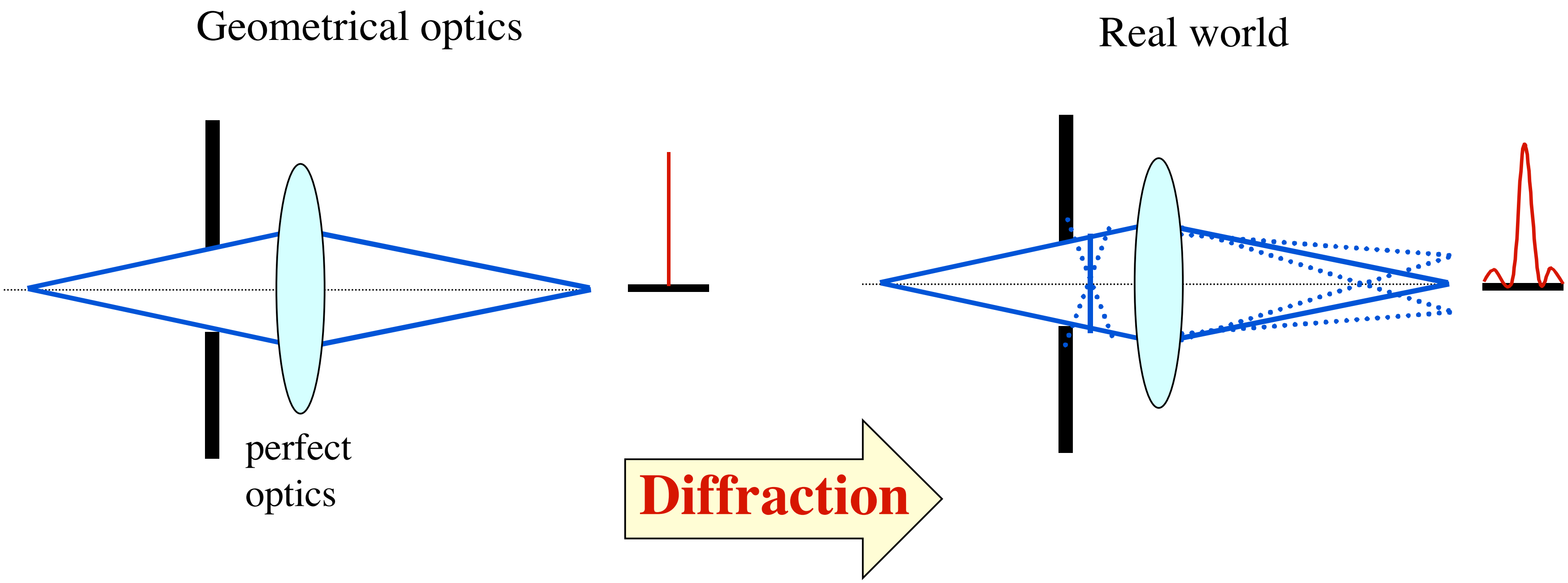
Point Spread Function (PSF)

$$PSF = \left| FT\left(P(x, y)\right)\right|^2$$

The Point Spread Function, or PSF, is the image that an optical system forms of a point source.

The point source is the most fundamental object, and forms the basis for any complex object.

Point Spread Function (PSF)



Geometrical optics

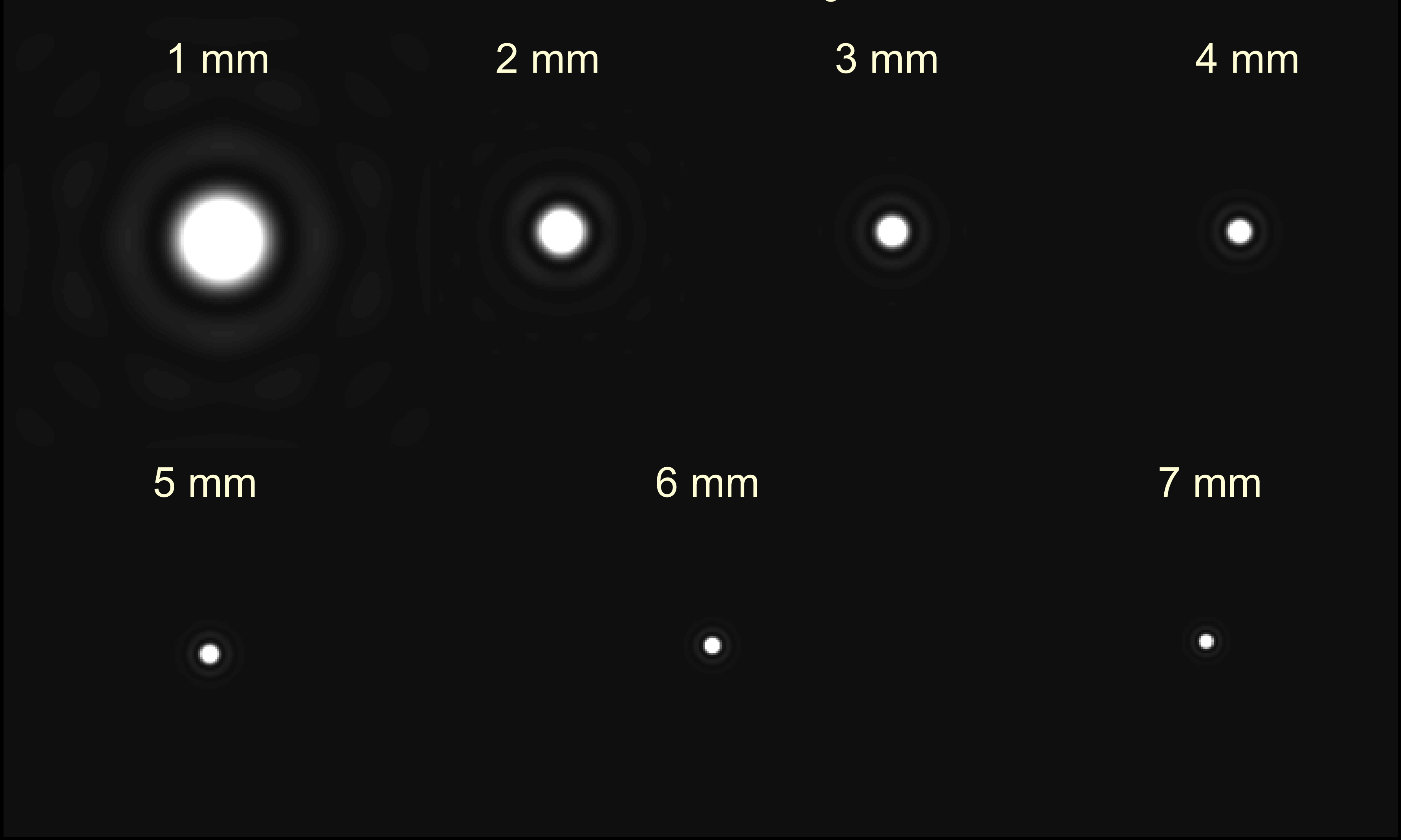
Real world

perfect optics

Diffraction

The PSF for a perfect optical system is the Airy disc, which is the Fraunhofer diffraction pattern for a circular pupil.

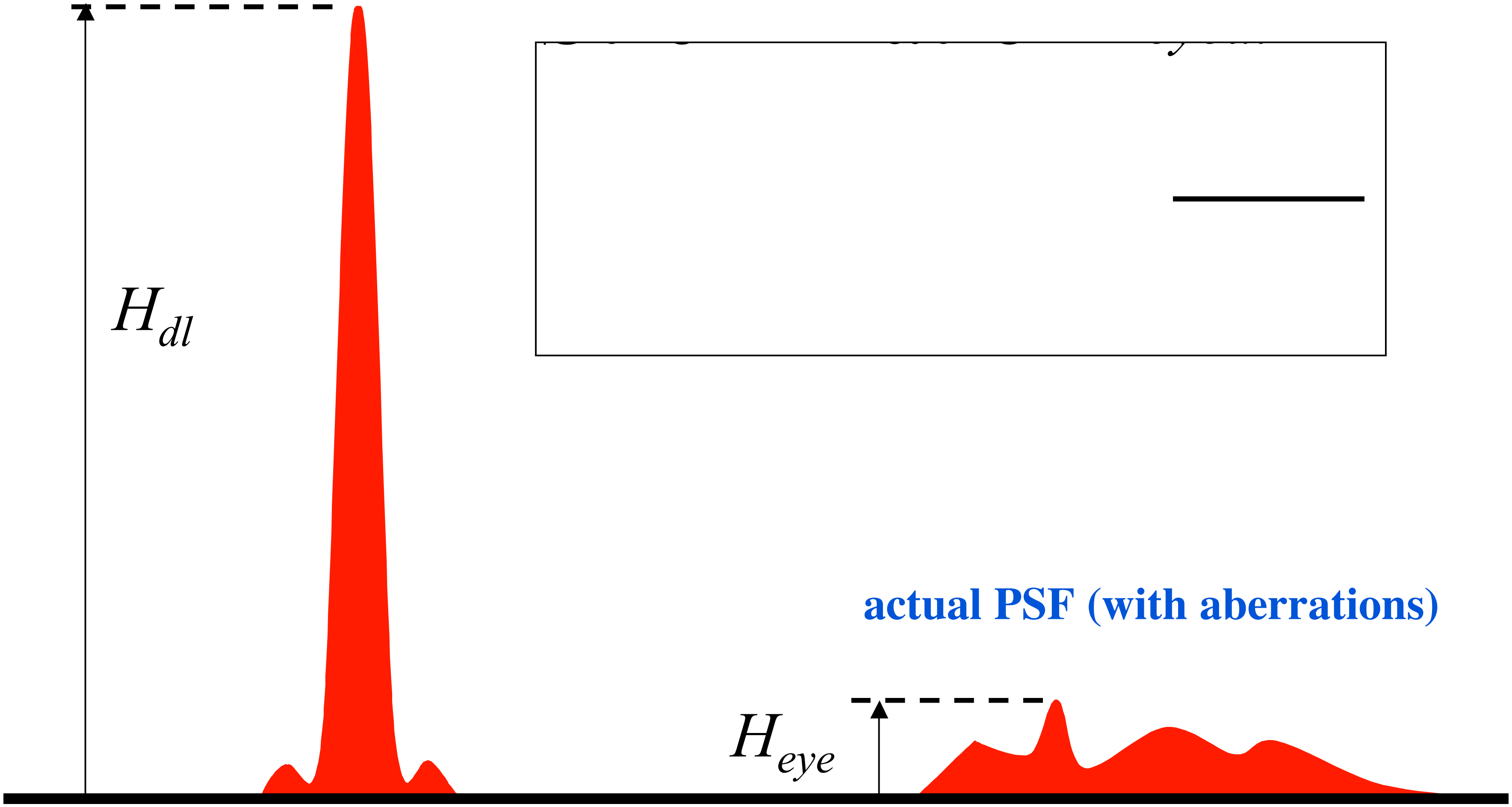
Point Spread Function vs. Pupil Size
Perfect Eye



Point Spread Function vs. Pupil Size
Typical Eye

1 mm
2 mm
3 mm
4 mm
5 mm
6 mm
7 mm

Strehl Ratio



diffraction-limited PSF (with no aberrations)

actual PSF (with aberrations)

Image Convolution

[illegible]

FT^{-1} [FT($PSF(x,y)$) °ø FT($O(x,y)$)] = $I(x,y)$

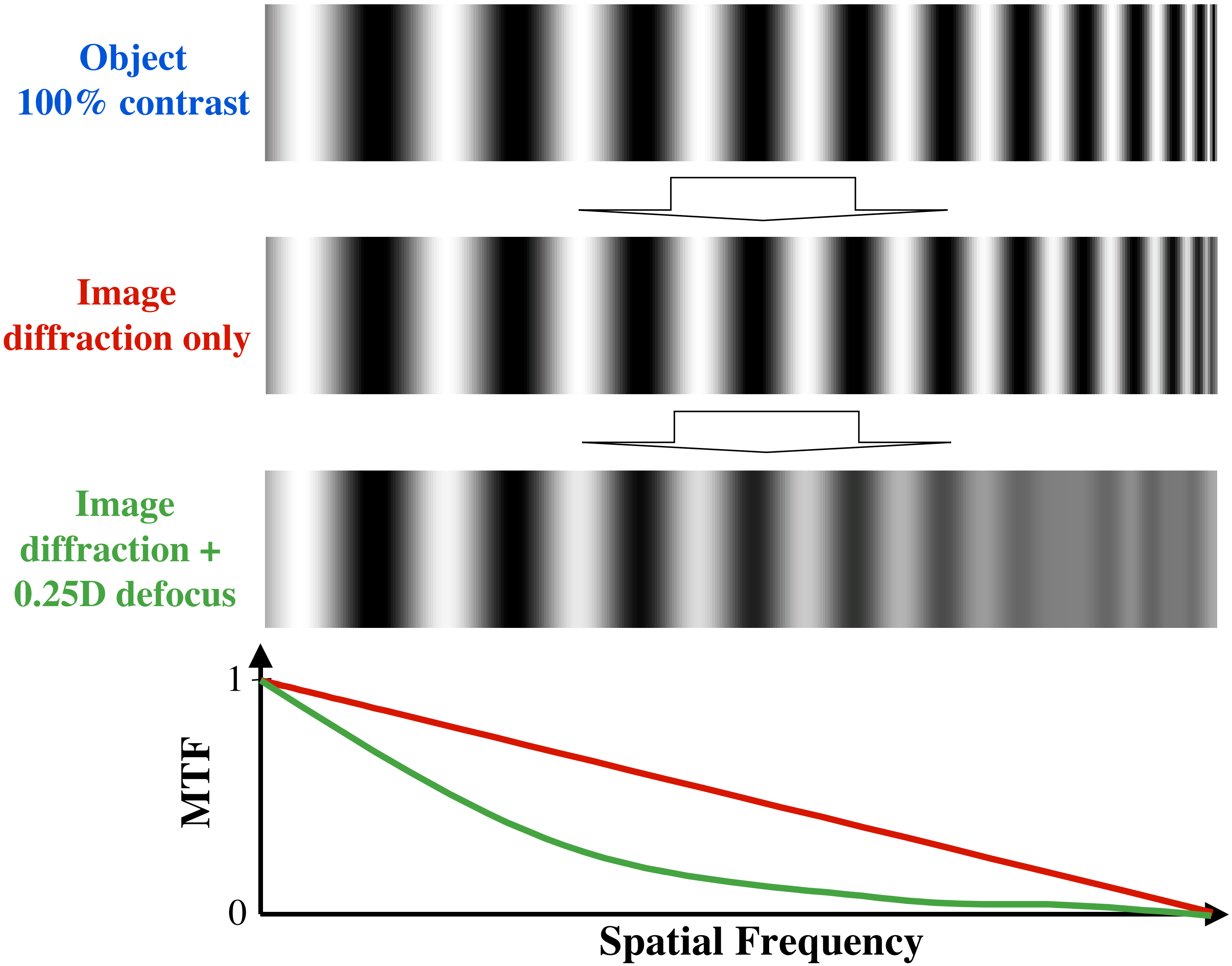
Modulation Transfer Function (MTF)

$$MTF\left(f_x, f_y\right) = \mathrm{Re}\left[FT\left(PSF\left(x, y\right)\right)\right]$$

The Modulation Transfer Function, or MTF, is a measure of the reduction in contrast from object to image.

The ratio of the image modulation to the object modulation at all spatial frequencies.

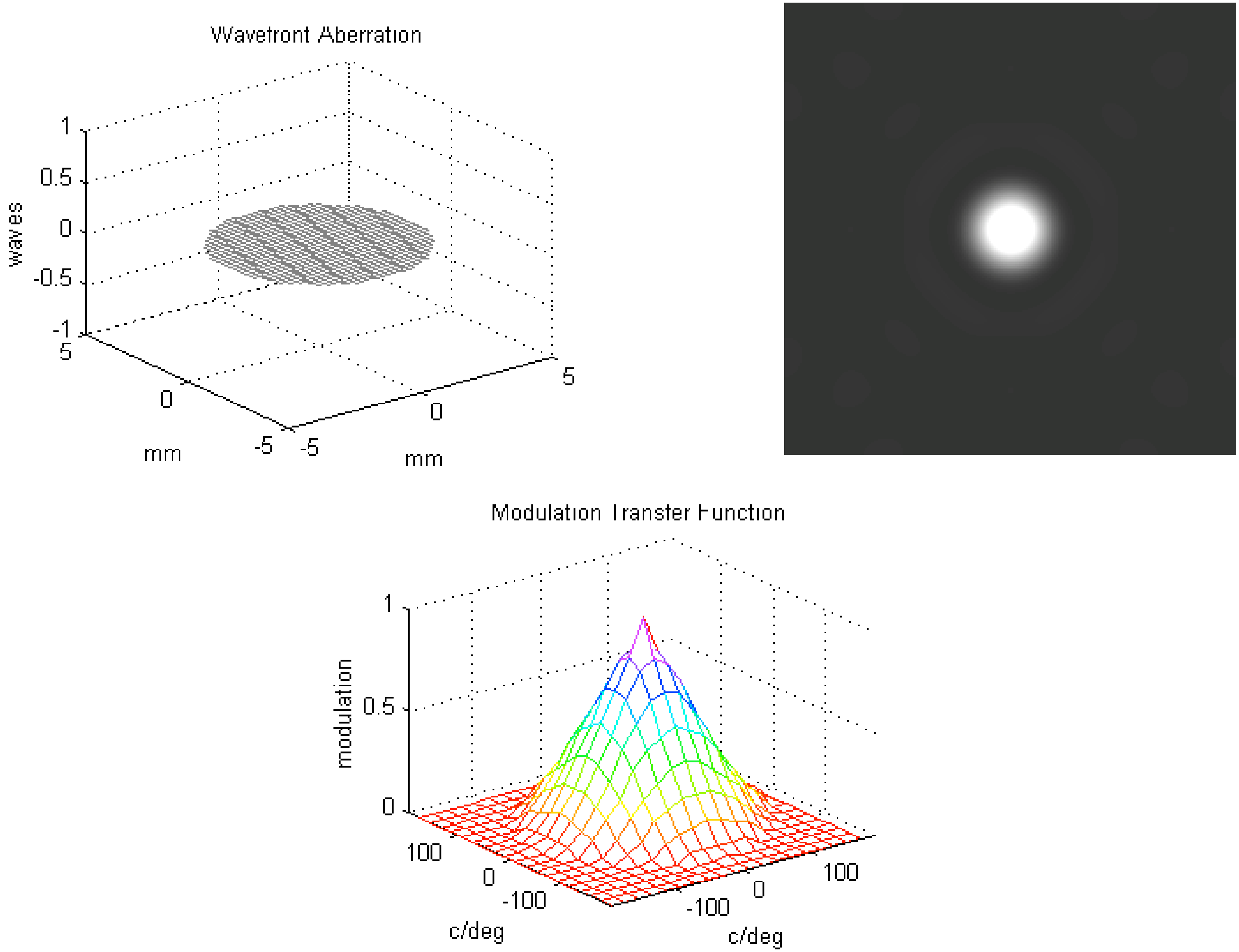
Modulation Transfer Function (MTF)

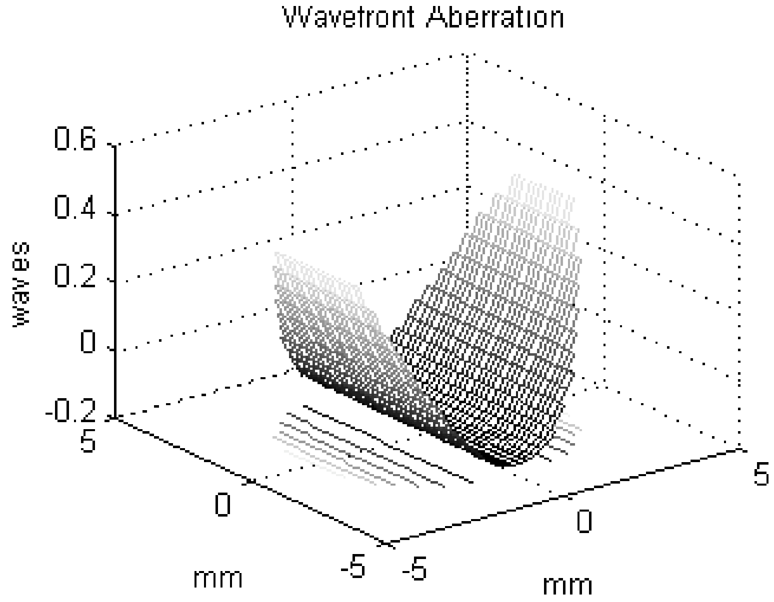


Object
100% contrast

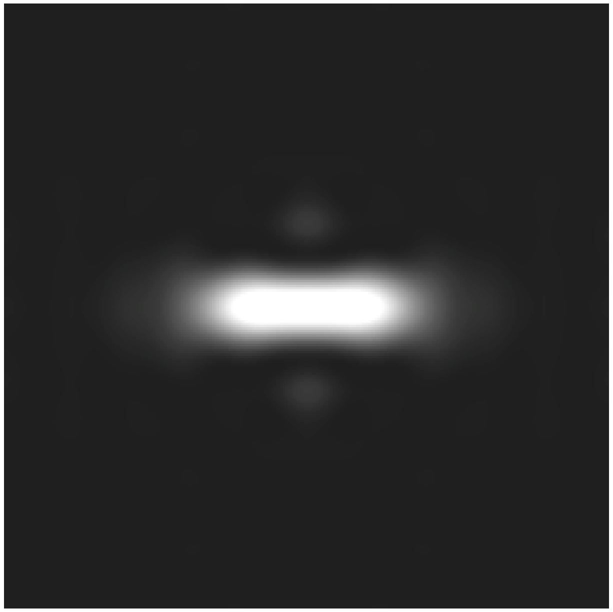
Image
diffraction only

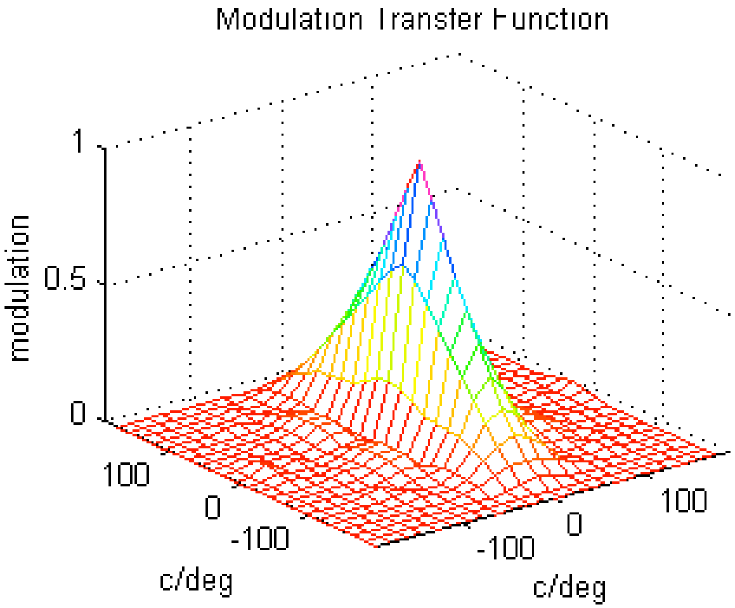
Image
diffraction +
0.25D defocus


Wavefront Aberration
waves
1
0.5
0
-0.5
-1
5
0
-5
mm
-5
0
5
mm
Modulation Transfer Function
modulation
1
0.5
0
100
0
-100
c/deg
-100
0
100
c/deg

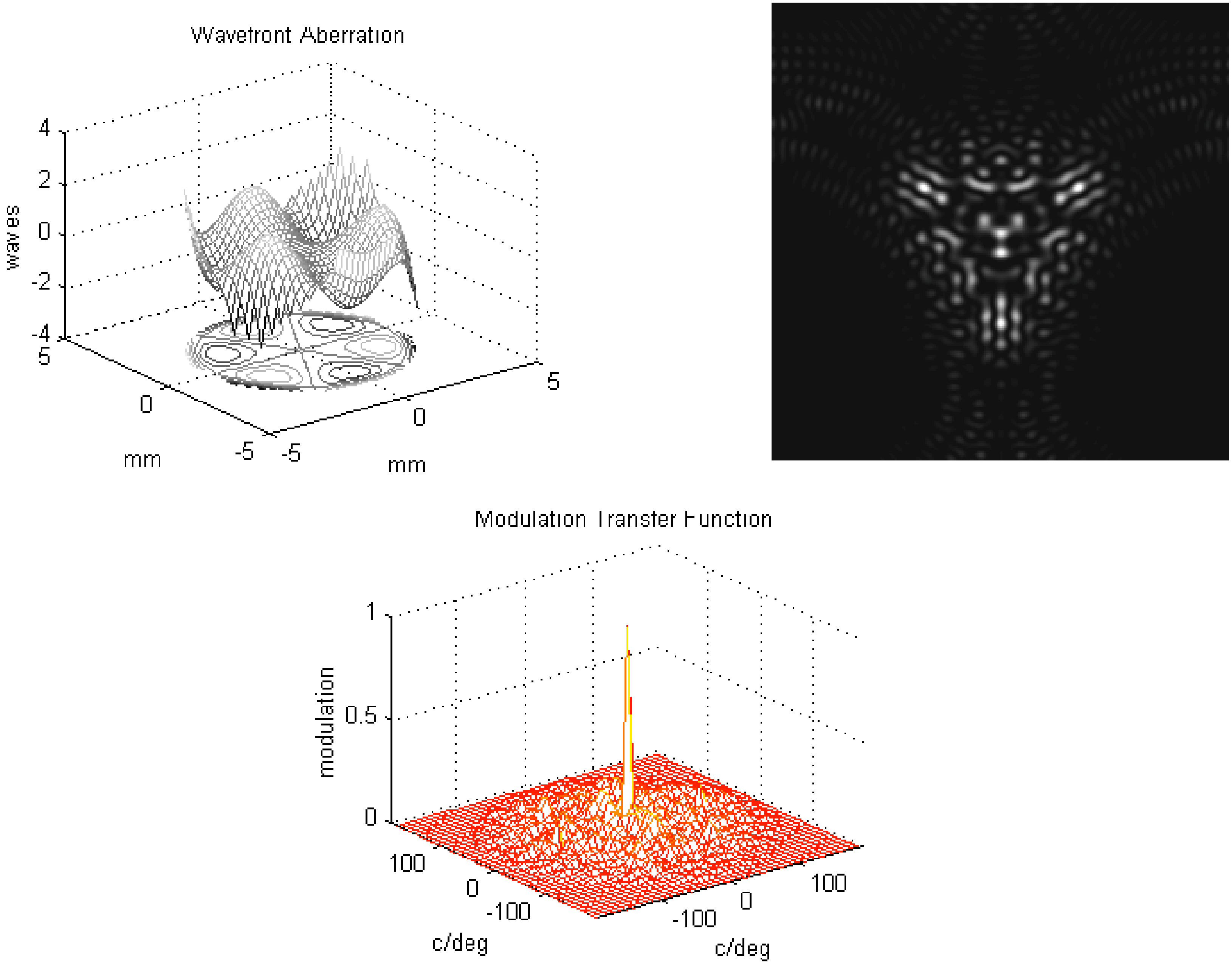


Wavefront Aberration





Modulation Transfer Function


Wavefront Aberration
waves
4
2
0
-2
-4
5
0
-5
mm
-5
0
5
mm
Modulation Transfer Function
modulation
1
0.5
0
100
0
-100
c/deg
-100
0
100
c/deg

Optics Simulations
- Have fun!!! -

Fundamental Optics

- http://webphysics.ph.msstate.edu/javamirror/java/light.htm

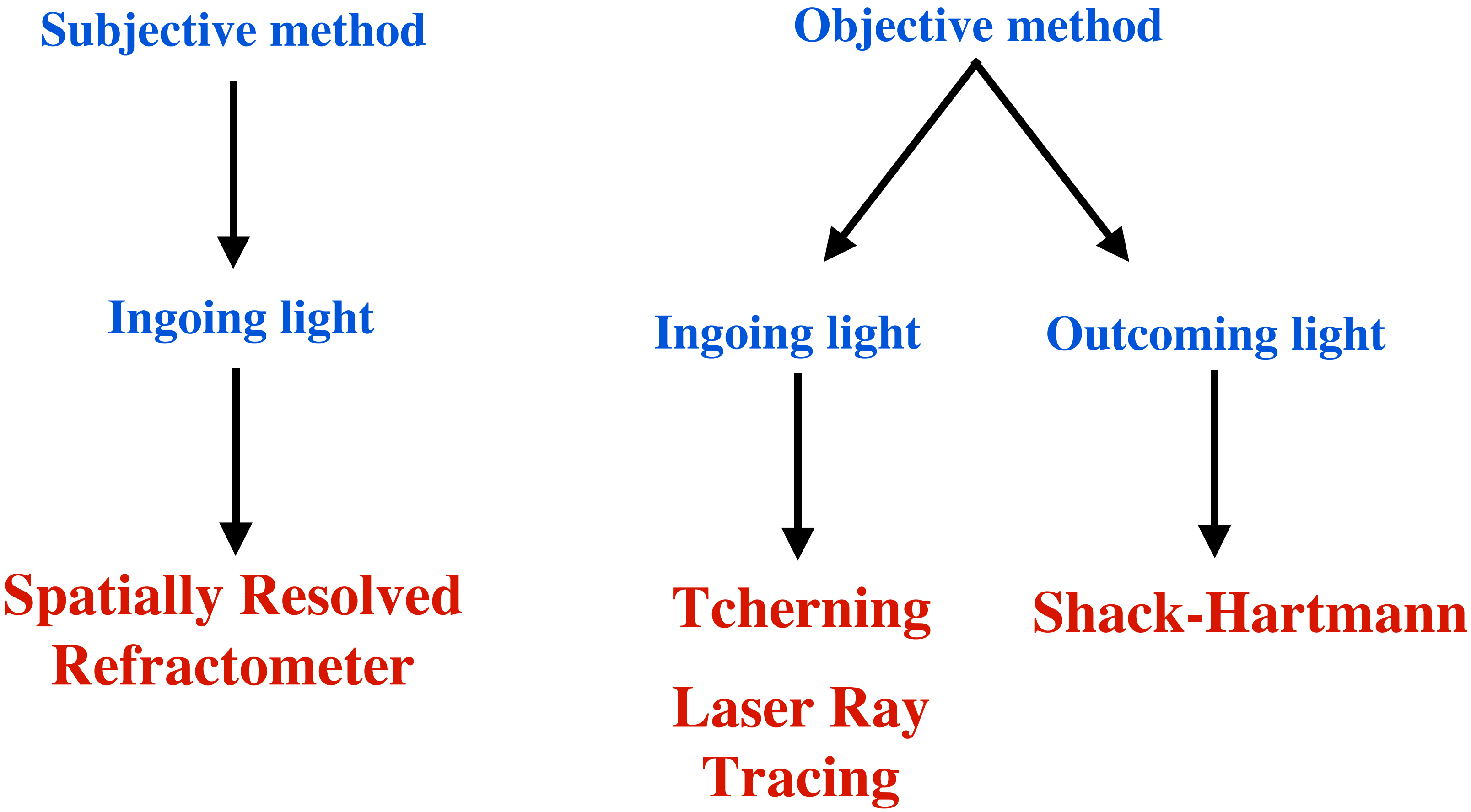
Wavefront Theory and Fourier Optics

- http://www.optics.arizona.edu/jcwyant/math.htm

How Can We Measure These Aberrations Of The Eye?

Different types of wavefront sensors to measure ocular aberrations

Wavefront sensors for the eye



Subjective method

Ingoing light

Spatially Resolved Refractometer

Objective method

Ingoing light

Tcherning

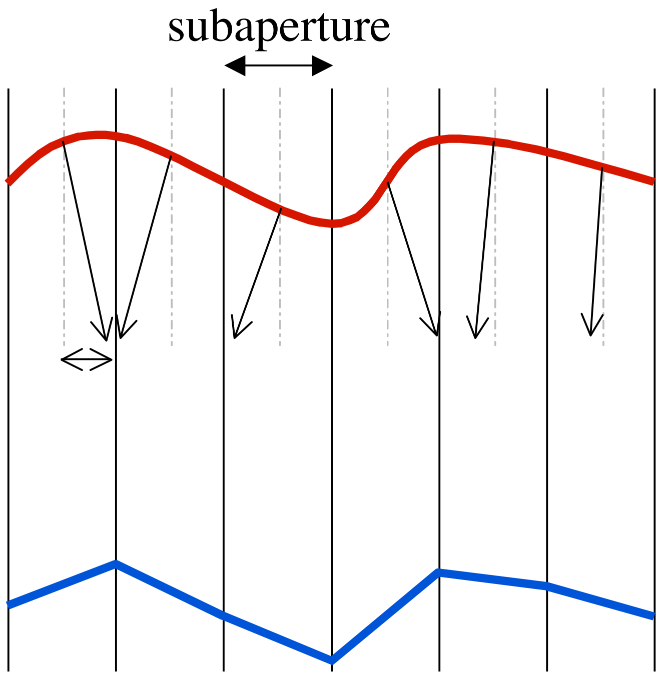
Laser Ray Tracing

Outcoming light

Shack-Hartmann

Measurement principle of wavefront sensors for the eye

Measurement of <u>wavefront slope (1st derivative of wavefront)</u> averaged over each subaperture on the pupil



Original wavefront
***W*(*x*,*y*)**

Spot displacement
Δd_x, _d_y

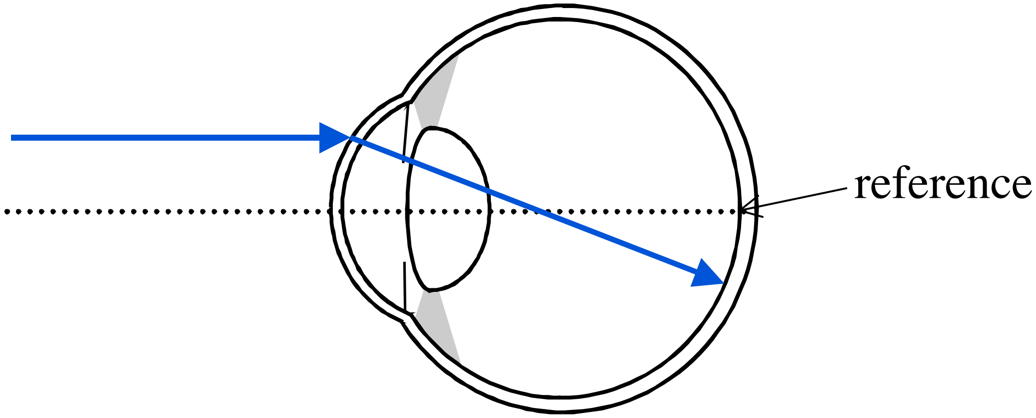
Averaged wavefront slope

$$\frac{\partial W(x,y)}{\partial x} = k \cdot \Delta d_x$$

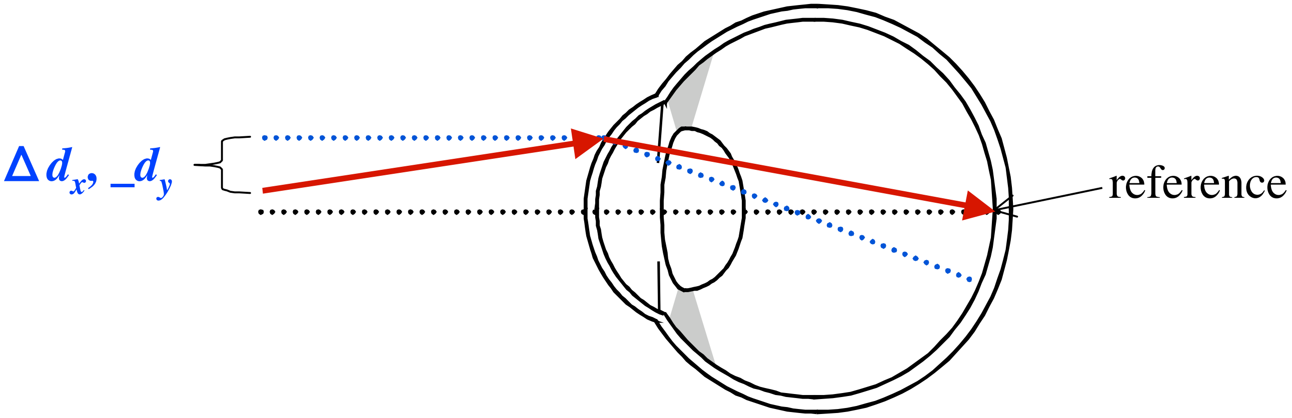
$$\frac{\partial W(x,y)}{\partial y} = k \cdot \Delta d_y$$

Spatially Resolved Refractometer

Webb, Penney and Thompson (1992)

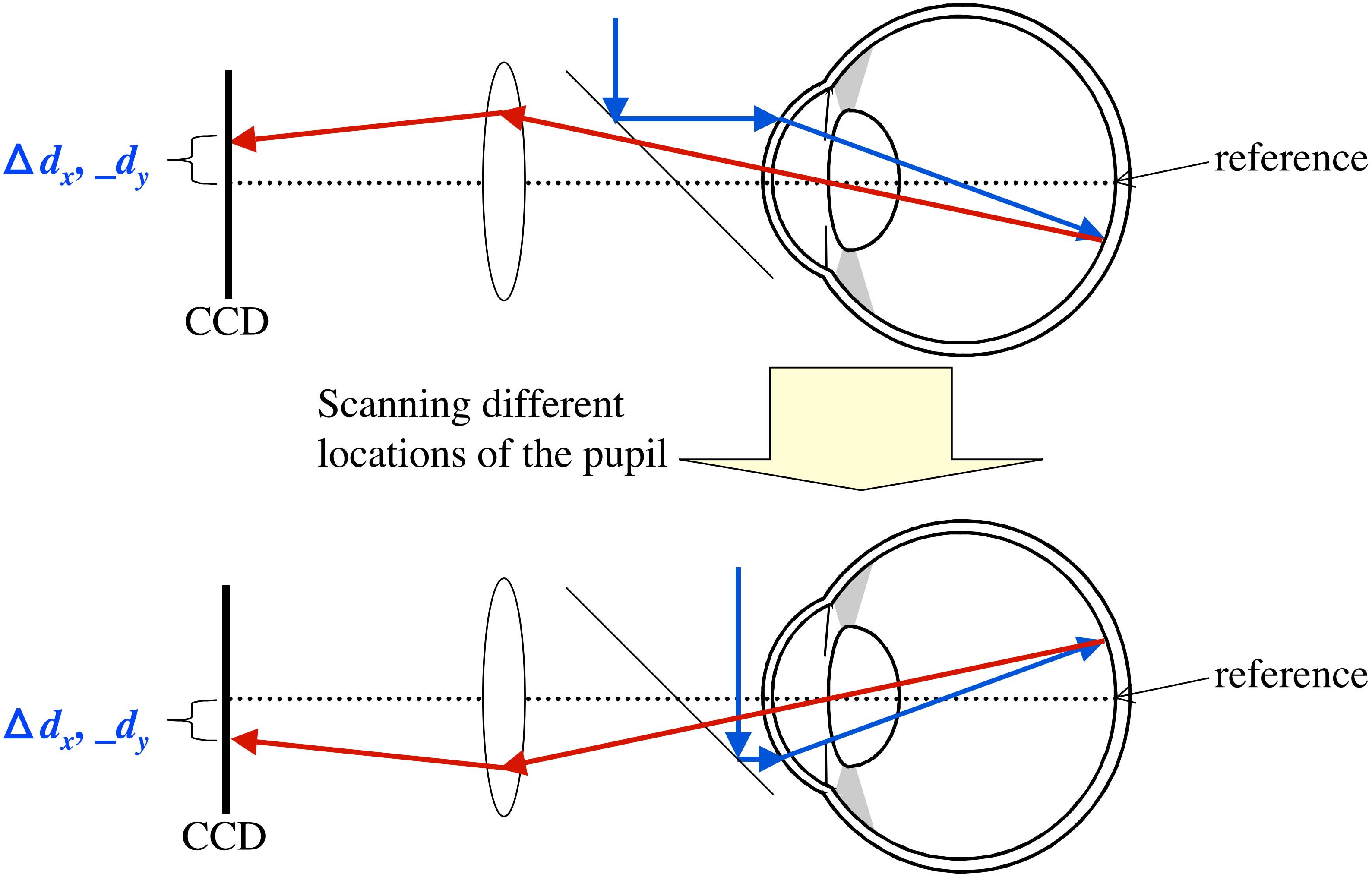


Subject adjusts the incident angle of light until retinal spot intersects reference spot.



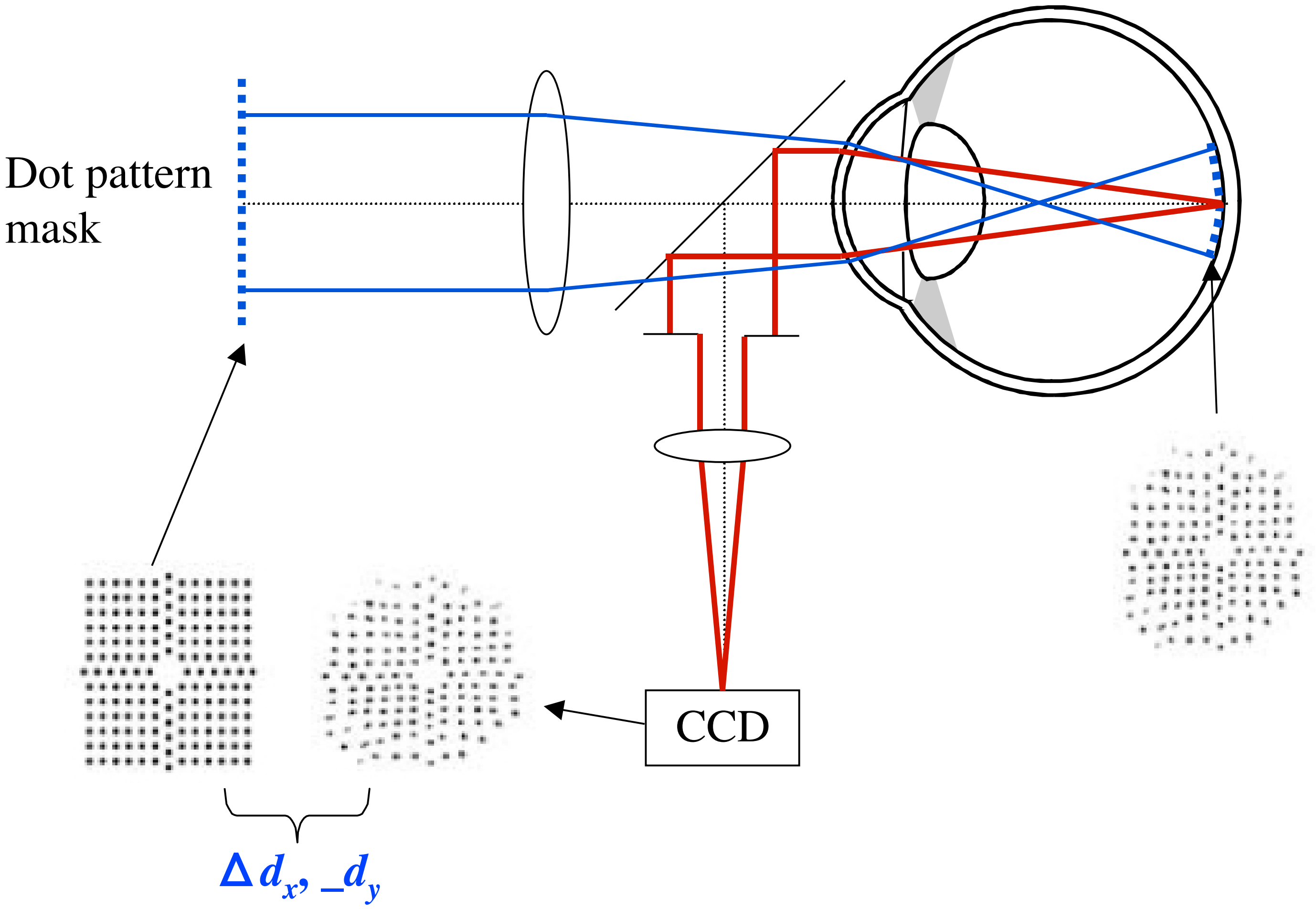
Laser Ray Tracing

Navarro & Losada (1997), Molebny et al. (1997)



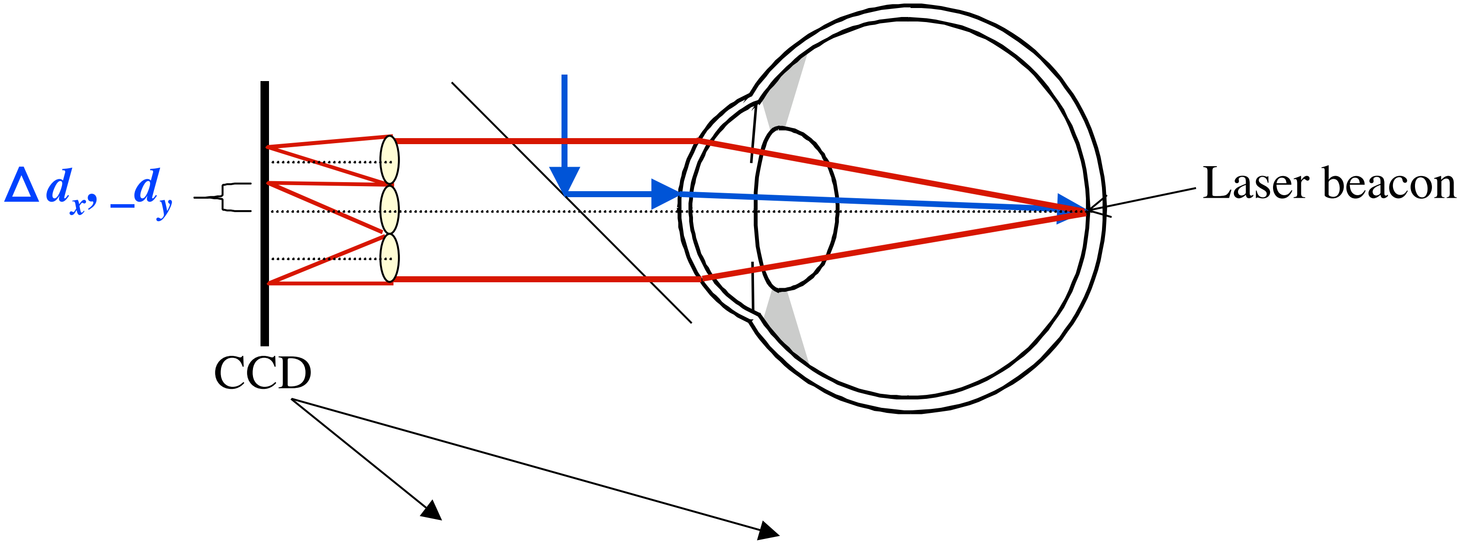
Tcherning Aberroscope

Tscherning (1894)

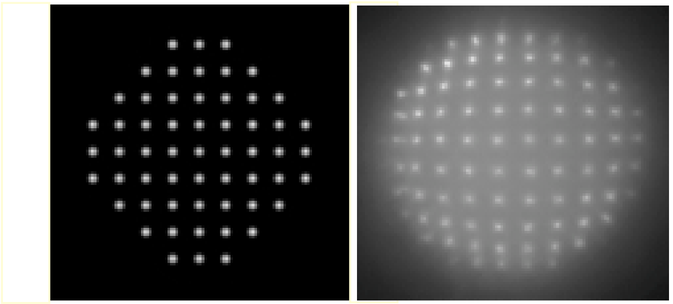


Shack-Hartmann wavefront sensor

Liang, Grimm, Goelz, and Bille (1994), Liang and Williams (1997)



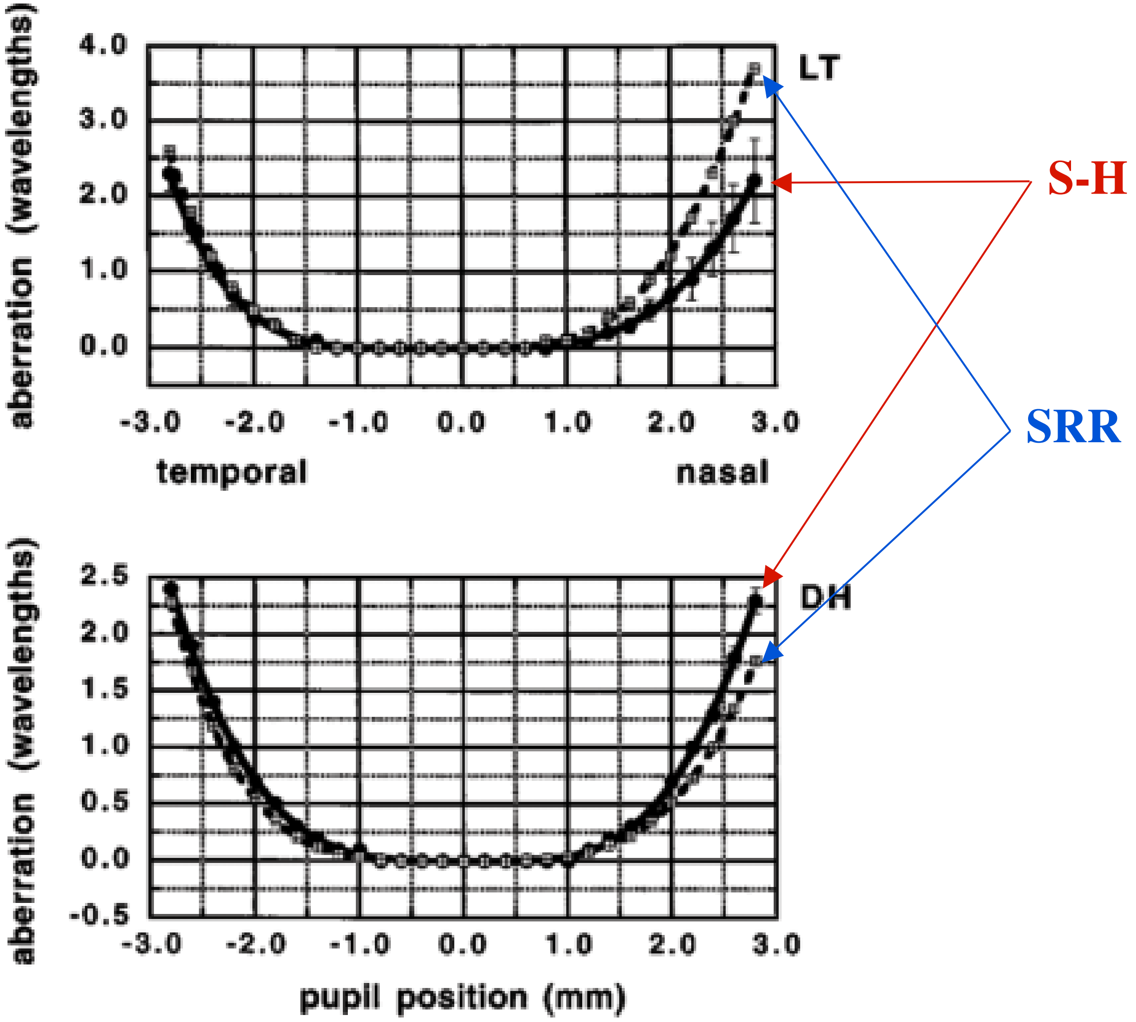
Perfect eye **Real eye**



Comparison of wavefront measurements using different wavefront sensors

Shack-Hartmann vs Spatially Resolved Refractometer (Objective vs Subjective methods)

Salmon et al., J. Opt. Soc. Am. A, 15 (1998)

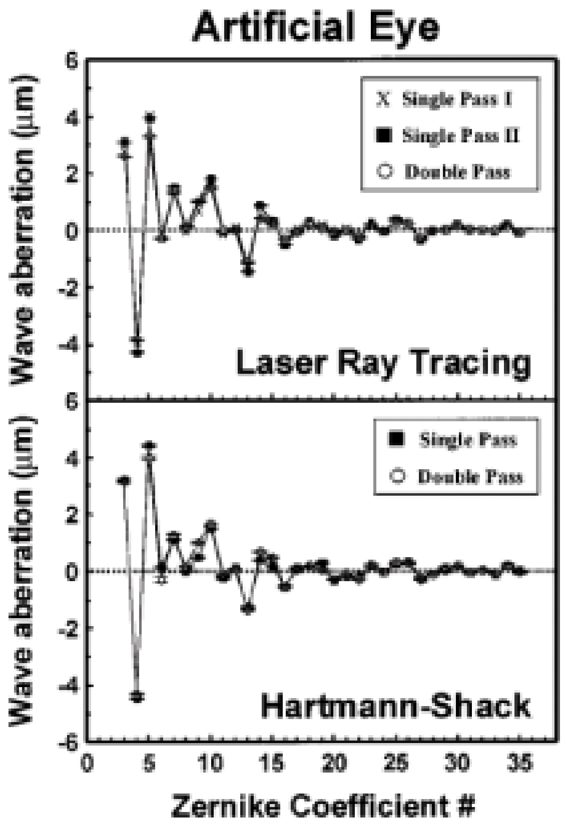


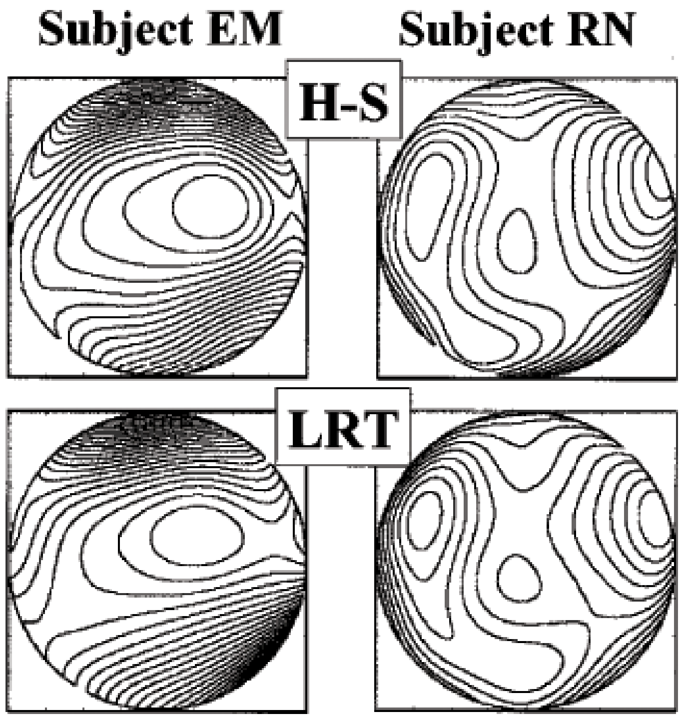
S-H

SRR

Shack-Hartmann vs Laser Ray Tracing (Outcoming vs Ingoing light)

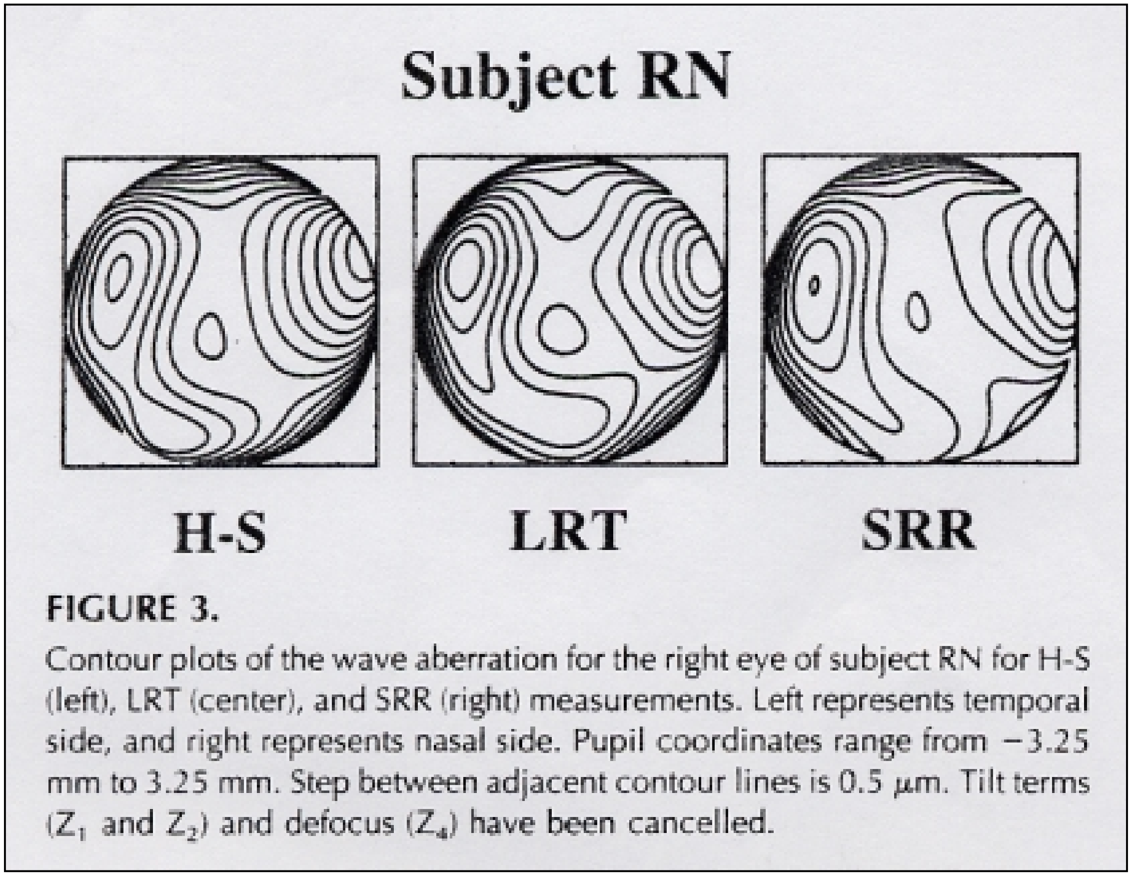
Moreno-Barriuso and Navarro, J. Opt. Soc. Am. A, 17 (2000)





Shack-Hartmann vs Laser Ray Tracing vs Spatially Resolved Refractometer

Moreno-Barriuso et al., ***Optometry and Vision Science*****, 78 (2001)**



Subject RN

H-S **LRT** **SRR**

FIGURE 3.

Contour plots of the wave aberration for the right eye of subject RN for H-S (left), LRT (center), and SRR (right) measurements. Left represents temporal side, and right represents nasal side. Pupil coordinates range from −3.25 mm to 3.25 mm. Step between adjacent contour lines is 0.5 µm. Tilt terms (Z_1 and Z_2) and defocus (Z_4) have been cancelled.

Thank you!